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AMENABLE CORRESPONDENCES AND APPROXIMATION PROPERTIES FOR VON NEUMANN ALGEBRAS

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We introduce the notion of amenable equivalence between von Neumann algebras, and study some approximation properties which remain invariant by this relation. We show for instance that the constant $\Lambda(M)$ associated with a von Neumann algebra M when considering the weak* completely bounded approximation property is an invariant for this equivalence relation. As an example, let α be an amenable action of a locally compact group G on a von Neumann algebra M; then the crossed product $M \rtimes_\alpha G$ is amenably equivalent to M. Another example is obtained by considering a pair $G_1 \subset G$ of locally compact groups such that the homogeneous space G/G_1 is amenable. Then the von Neumann algebras $W^*(G)$ and $W^*(G_1)$ generated by the left regular representations of G and G_1 respectively are amenably equivalent. Therefore, if moreover G is discrete, we get that G and G_1 are simultaneously weakly amenable with the same Haagerup's constants $\Lambda_G = \Lambda_{G_1}$.

Introduction

Given a pair $M \subset N$ of von Neumann algebras, one finds many situations where there exists a norm one projection E from N onto M, and one may ask what properties of N are automatically inherited by M in this case, the most well-known example being the amenability of N. In this paper we will see that other approximation properties such as the weak* completely bounded approximation property ([**Haa4**], [**C-H**]), or the σ-weak approximation property of [**K**] are also preserved. Their common feature is the approximation of the identity map of N by appropriate σ-weakly continuous bounded maps, and the main problem is that the norm one projection E is not σ-weakly continuous in general.

The existence of E follows easily from the existence of a net $(\phi_i)_{i\in I}$ of σ-weakly continuous completely positive contractions $\phi_i : N \to M$ such that $\lim_i \phi_i(x) = x$ σ-weakly for all $x \in M$. The converse is very likely true, but we are not able to prove it in full generality (see Cor. 3.9). At least we show that, when there exists a norm one projection E from N onto M, we may