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ON H^P-SOLUTIONS OF THE BEZOUT EQUATION

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We obtain a sufficient condition on bounded holomorphic functions g_1, g_2 in the unit disk for the existence of f_1, f_2 in the Hardy space H^p such that $1 = f_1g_1 + f_2g_2$. The sharpness of this condition is also studied.

1. Let $\mathbb{D}$ be the unit disk in the complex plane, $\mathbb{T}$ its boundary. For $1 \leq p \leq \infty$, H^p denotes the Hardy-space of holomorphic functions in $\mathbb{D}$ such that

$$\|f\|_p = \sup_r \left(\frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right)^{1/p} < +\infty \qquad p < \infty$$

$$\|f\|_\infty = \sup_{|z|<1} |f(z)| .$$

It is well-known ([**7**, p. 57]) that if $f \in H^p$, the non-tangential maximal function

$$Mf(e^{i\theta}) = \sup\{|f(z)|;\ z \in \Gamma(\theta)\}$$

$\Gamma(\theta)$ being the Stolz angle with vertex $e^{i\theta}$, belongs to $L^p(\mathbb{T})$.

In this paper, given $g_1, g_2 \in H^\infty$, we study the Bezout equation $1 = f_1g_1 + f_2g_2$. Concretely, we are interested in knowing the precise condition on g_1, g_2 so that solutions $f_1, f_2 \in H^p$ exist.

If $|g|^2 = |g_1|^2 + |g_2|^2$, $|f|^2 = |f_1|^2 + |f_2|^2$, it follows from $1 = f_1g_1 + f_2g_2$ that $1 \leq |f|\,|g|$ and hence

$$\text{(C)} \qquad M(|g|^{-1}) \in L^p(\mathbb{T}) .$$

It can be easily seen that this condition is sufficient if g_1 or g_2 is an interpolating Blaschke product. Nevertheless, we show in Section 2 that it is not sufficient in general. In fact for each $\varepsilon > 0$ we exhibit $g_1, g_2 \in H^\infty$ such that $M(|g|^{-2+\varepsilon}) \in L^p(\mathbb{T})$ but no H^p solutions exist.

In Section 3 we obtain a general sufficient condition implying in particular the following:

Theorem 1. *Assume that for some $\varepsilon > 0$*

$$M\left(|g|^{-2}\,|\log|g||^{2+\varepsilon}\right) \in L^p(\mathbb{T}) .$$