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NEVANLINNA'S COEFFICIENTS AND DOUGLAS ALGEBRAS

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Some relations between Douglas algebras and coefficients appearing in Nevanlinna's matrix parametrization of the solutions of the Nevanlinna Pick interpolation problem are studied.

1. Introduction.

Let U denote the analytic functions bounded by one in $\mathbb{D} = \{z : |z| < 1\}$. Given a sequence $\{z_n\} \subset \mathbb{D}$, we consider the classical Nevanlinna Pick interpolation problem

$$\text{(NP)} \qquad f(z_n) = w_n, \qquad n = 1, 2, \dots, \qquad f \in U.$$

If this problem has more than one solution, R. Nevanlinna [4] found analytic functions P, Q, R and S such that the set of all solutions is given by

$$E = \left\{ \frac{P - Qw}{R - Sw}, \quad w \in U \right\}. \tag{1.1}$$

The functions P, Q, R and S are unique subject to the normalization $S(0) = 0$ and $PS - RQ = \pi$, where

$$\pi(z) = \prod_n \frac{|z_n|}{z_n} \frac{z_n - z}{1 - \overline{z}_n z}$$

is the Blaschke product corresponding to $\{z_n\}$.

While the funcions P, Q, R and S arose from classical function theory, it turns out that they are also connected with more recent developments. It is part of Nevanlinna's theory that the functions $P/R, Q/R, S/R$ and $1/R$ belong to U and are linked with π in many ways. (See Lemma 1.)

Suppose (NP) has a solution f_0 satisfying $\sup\{|f_0(z)|,\ z \in D\} < 1$. Our main result is that then $P/R, Q/R, S/R$ and $1/R$ all belong to a certain subalgebra of H^∞ depending only on π which we shall denote by CDA_π. This algebra is part of the theory of Douglas algebras through the work of S.Y. Chang and D.E. Marshall ([1], [2?]). Our results in particular answer