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IRREDUCIBILITY AND DIMENSION THEOREMS FOR FAMILIES OF HEIGHT 3 GORENSTEIN ALGEBRAS

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We show that the family of graded Gorenstein Artin algebras of height 3 with a fixed Hilbert function is irreducible, and we prove some dimension theorems about these families.

0. Introduction.

In Chapter 1 we show that when a set $D = (Q, P)$ of generator and relation degrees is given, $Q = \{q_1, \ldots, q_u\}$ and $P = \{p_1, \ldots, p_u\}$, the family Gor_D of Gorenstein algebras $A = R/J$ with J having this set of generator and relation degrees is irreducible. We show that Gor_D is the image of an algebraic map from a dense open set in a product of affine spaces. This depends on Buchsbaum and Eisenbud's structure theorem for height 3 Gorenstein ideals [**BE1**], which is discussed in Chapter 2.

In Chapter 2 we show that when T is fixed, the family Gor_T of all Gorenstein algebras with Hilbert function T is irreducible. We show this by giving an explicit deformation of an ideal with degree set D to an ideal with a smaller degree set D' consistent with T. The minimal set D_{min} of generator and relation degrees given T is unique, and we conclude in Theorem 2.7 that $\overline{Gor_D} \supset Gor_{D'}$ for $D' \supset D$, and therefore $Gor_T = \overline{Gor_{D_{min}}}$, the Zariski closure of $Gor_{D_{min}}$ inside Gor_T. We give a method for determining the alternating matrix whose pfaffians generate the ideal with the smaller degree set, and show that it is Gorenstein of height 3. We conclude that whenever an ideal J determining T is generated by more than the minimum number needed for T, it can be deformed to an ideal with fewer generators.

We again work from the perspective of a fixed Hilbert function T in Chapter 3 to determine the maximum number of generators an ideal determining T may have. This uses the combinatorial data described in [**BE1**] and [**St1**], specifically the conditions on a sequence $\{r_1, r_2, \ldots, r_u\}$ of integers, which we call *diagonal degrees*, that can occur as the differences in a degree set D defining a Gorenstein algebra with Hilbert function T. We determine the maximum number of generators possible for a ideal determining a given Hilbert function, and we give an explicit example of a matrix whose pfaffians generate this number for any given T. There is a lattice structure we can