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ON THE MINIMAL FREE RESOLUTION OF GENERAL EMBEDDINGS OF CURVES

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Here we study the minimal free resolution of general embeddings in $\mathbf{P}^n$ of genus g curves with general moduli. We prove that if p is an integer with, roughly, $g \leq n^2/(2p+2)$, then the embedding has the property N_p, i.e., the first p pieces of the resolution are as simple as possible.

We work over an algebraically closed field. Let C be a smooth curve embedded in $\mathbf{P}^n$. We are interested in the minimal free resolution of C. Here we will consider the case in which the curve has general moduli and the embedding is general. Recall the following definition ([**5**], [**6**]).

Definition 0.1. Let $C \subset \mathbf{P}^n$ be a reduced curve; fix an integer $p \geq 1$; C satisfies the property N_p if C is arithmetically Cohen - Macaulay and for every integer i with $1 \leq i \leq p$ the i^{th}-sheaf appearing in the minimal free resolution of the homogeneus ideal of C is the direct sum of line bundles of degree $-i-1$.

For instance if we say that N_0 means "C is arithmetically Cohen-Macaulay", then N_1 means that the curve C is N_0 and its homogeneous ideal is generated by quadrics. Furthermore, if $p > 0$, then N_p implies N_{p-1}.

In this paper, using degeneration techniques, we will prove the following results (Theorems 0.2 and 0.3).

Theorem 0.2. *Fix an integer $p \geq 1$. For every integer u, set:*

$$\alpha_p(u) := (u^2)/(2p+2) - (u/2). \tag{1}$$

Fix an integer $n \geq 3$ with $n \geq p+1$, and set:

$$G_p(n) := \alpha_p((p+1)[n/(p+1)]) \tag{2}$$

where $[y]$ is the greatest integer $\leq y$. Then for every integer $g \leq G_p(n)$ the general linearly normal non special curve $C \subset \mathbf{P}^n$ with $p_a(C) = g$ and $\deg(C) = g+n$ satisfies the property N_p.

Note that $G_p(n)$ has order $(n^2)/(2p+2)$ and hence $d := g+n$ is usually much smaller than $2g+p$ if n is much larger than p.