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THE WEYL QUANTIZATION OF POISSON $SU(2)$

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In this paper, we consider the problem of quantizing the canonical multiplicative Poisson structure on $SU(2)$ by C*-algebraic deformation, a notion introduced by Rieffel, and show that there is such a deformation which is also a coalgebra homomorphism. Parallel to the algebraic development of quantum group theory, Woronowicz successfully quantized the group structure of $SU(2)$ (and other groups) through deformation in the context of Hopf C*-algebras. It is known that there exists a C*-algebraic deformation quantization of the multiplicative Poisson structure on $SU(2)$ which is 'compatible' with Woronowicz's deformation (of the group structure) on the C*-algebra level. Although that deformation preserves the important symplectic leaf structure on $SU(2)$ in a natural way, it does not preserve the group struture in the sense that it is not a coalgebra homomorphism. We show that the Weyl transformation introduced by Dubois-Violette gives a different C*-algebraic deformation quantization which is compatible with Woronowicz's deformation and does preserve the group structure.

1. Introduction.

In recent years, there has been a fast growing interest in the theory of deformation quantization (initiated in [**Ge**], [**BFFLS**]), which fits nicely with the concept of non-commutative geometry [**C1**]. There are many papers in this area using different approaches, either analytic, algebraic, geometric, or physical, and some of them have an extensive reference list [**R5**], [**Wo2**], [**K**], [**Gi**], [**Dr**], [**We-X**]. We refer readers to these sources for a broad overview.

It is known [**Sh1**] that there exists a C*-algebraic deformation quantization, a concept introduced by Rieffel [**R1**], of the multiplicative Poisson structure on $SU(2)$ [**Lu-We**] which is compatible with Woronowicz's C*-algebraic deformation quantization of the group structure on $SU(2)$ [**Wo1**], in the sense that the C*-algebras obtained in these two processes are isomorphic. This result shows that on the algebra level, the multiplicative Poisson structure can be deformed in a way compatible with the deformed group structure of $SU(2)$ during quantization.