PACIFIC JOURNAL OF MATHEMATICS
Vol. 173, No. 2, 1996

INTERPOLATING BLASCHKE PRODUCTS GENERATE H^∞

JOHN GARNETT AND ARTUR NICOLAU

The algebra of bounded analytic functions on the open unit disc is generated by the set of Blaschke products having simple zeros which form an interpolating sequence.

Let H^∞ be the algebra of bounded analytic functions in the unit disc $\mathbb{D}$ and set

$$||f|| = \sup_{z \in \mathbb{D}} |f(z)|,$$

for $f \in H^\infty$. A *Blaschke product* is an H^∞ function of the form

$$B(z) = \prod_{\nu=1}^{\infty} \frac{-\overline{z_\nu}}{|z_\nu|} \frac{z - z_\nu}{1 - \overline{z_\nu} z}$$

with $\sum(1 - |z_\nu|) < \infty$. In [**5**] D.E. Marshall proved that H^∞ is the closed linear span of the Blaschke products: given $f \in H^\infty$ and $\varepsilon > 0$, there are constants $c_1, \dots, c_n$ and Blaschke products $B_1, \dots, B_n$ such that

$$(1) \qquad ||f + c_1 B_1 + \cdots + c_n B_n||_\infty < \varepsilon.$$

In fact, Marshall proved that the unit ball of H^∞ is the uniformly closed convex hull of the set of Blaschke products (including $B \equiv 1$).

A Blaschke product $B(z)$ is called an *interpolating Blaschke product* if

$$(2) \qquad \inf_{\nu} (1 - |z_\nu|^2) \, |B'(z_\nu)| = \delta_B > 0,$$

because of the Carleson theorem that (2) holds if and only if every interpolation problem

$$f(z_\nu) = w_\nu, \qquad \nu = 1, 2, \dots,$$

for $\{w_\nu\} \in l^\infty$, has a solution $f \in H^\infty$. Although the interpolating Blaschke products comprise a small subset of the set of all Blaschke products, they play a central role in the theory of H^∞. See the last three chapters of [**3**]. The theorem in this paper helps explain why interpolating Blaschke products are so important in that theory.

Theorem. *H^∞ is the closed linear span of the interpolating Blaschke products.*

In other words, (1) is true with the additional proviso that each of $B_1, \dots,$ B_n is an interpolating Blaschke product.