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RIGIDITY OF ISOTROPIC MAPS

FERNANDO CUKIERMAN

We consider a rigidity question for isotropic harmonic maps from a compact Riemann surface to a complex projective space. In the case of the projective plane, we prove that ridigity holds if the degree is small in relation to the genus. For a projective space of any dimension we obtain coarser results about rigidity and rigidity up to finitely many choices.

Introduction.

Let $f, g : X \to \mathbb{P}^r$ denote two isotropic harmonic maps from a compact Riemann surface to complex projective space. In this article we study whether from the isometry of f and g one may conclude their unitary equivalence.

Using Calabi's rigidity theorem, this question may be reduced to one in the algebraic category, involving certain curves of osculating spaces to a holomorphic curve. We obtain some rigidity results mostly by analyzing the quadrics containing those curves.

After recalling some definitions and basic facts, we show in §1 that our unitary question may be reduced to a projective one. Then in §2 we record some rigidity statements that follow easily from the use of projective invariants. In §3 and §4 we consider plane curves; we prove in (3.8) and (4.13) that ridigidy holds, roughly speaking, if the degree is small compared to the genus, providing a partial answer to a question posed by Quo-Shin Chi [**C**].

Motivated by the method of proof of Theorem (3.8), in §4 we begin a study of the ideal of associated curves and of the curves $f^{\langle k \rangle}(X)$ introduced in §1. This is related to some aspects of Brill-Noether theory that we plan to pursue in another article.

§1.

(1.1) We consider harmonic maps $X \to \mathbb{P}^r$ from a compact Riemann surface to complex projective space. One way of constructing such harmonic maps is the following: start with a holomorphic non-degenerate $f : X \to \mathbb{P}^r$ and, using the Fubini-Study metric of $\mathbb{P}^r$, construct a Frenet frame $f = f_0, f_1, \dots, f_r$. The maps $f_i : X \to \mathbb{P}^r$ are harmonic, and, for the purpose of this paper, harmonic maps obtained by this process will be called <u>isotropic</u> maps. We refer to [**EW**] for definitions and details on this construction.