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KK-GROUPS OF TWISTED CROSSED PRODUCTS BY GROUPS ACTING ON TREES

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An exact sequence of Pimsner for KK-groups of crossed products of C^*-algebras by locally compact groups acting on trees is generalized to the case of crossed products twisted by a circle-valued cocycle. The exact sequence is applied to the case of free products of twisted group C^*-algebras. In particular, the K-groups of the free product of two matrix algebras is computed.

1. Introduction.

The problem motivating this work was that of determining the K-theory of the free product $M_n *_{\mathbb{C}} M_k$ where M_n is the C^*-algebra of n by n complex-entried matrices. Specifically, the desire was to show that the K-groups of this particular free product satisfy the following repeating exact sequence.

$$\longrightarrow K_j(\mathbb{C}) \longrightarrow K_j(M_n) \oplus K_j(M_k) \longrightarrow K_j(M_n *_{\mathbb{C}} M_k) \longrightarrow .$$

This exact sequence is a special case of an exact sequence for amalgamated products conjectured by J. Cuntz in [**Cu2**]. It was proved by Cuntz that if $A *_C B$ is an amalgamated product of C^*-algebras subject to the condition that there are retractions from A and B onto C (homomorphisms which restrict to the identity on C) then the following repeating sequence is exact.

$$\longrightarrow K_j(C) \longrightarrow K_j(A) \oplus K_j(B) \longrightarrow K_j(A *_C B) \longrightarrow .$$

This conjecture that the above sequence holds for arbitrary amalgamated products has been verified for a variety of other cases since. For example, it follows from Pimsner's exact sequence for KK-groups of crossed products by groups acting on trees [**Pi**] that the above sequence is exact if $A = C^*(G_1)$, $B = C^*(G_2)$, and $C = C^*(H)$ where H is a discrete subgroup of the discrete groups G_1 and G_2. The conjecture has also been proved for the case where $A = M_n$, $C = \mathbb{C}$, and B is any unital C^*-algebra which has a retraction onto $\mathbb{C}$ in [**McC2**] (this result is primarily due to J. Cuntz). This conjecture is also true in many cases for reduced free products in the sense of D. Avitzour [**Av**] (see [**McC2**]). In [**McC2**] the author was able to exploit some of the