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# PREPOLAR DEFORMATIONS AND A NEW LÊ-IOMDINE FORMULA

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**In this short paper, we wish to describe a class of deformations of affine hypersurface singularities. These deformations, which we call *prepolar deformations*, are the ones which we found to be important in our work on the Lê numbers of a hypersurface singularity. However, the results of this paper are not directly related to our work on the Lê numbers.**

## Introduction.

The main result of this paper is to use prepolar deformations to put lower bounds on the top Betti number of the Milnor fibre of a non-isolated hypersurface singularity. These lower bounds are a corollary of a generalization of the formula of Lê and Iomdine.

Let $\mathcal{U}$ be an open neighborhood of the origin in $\mathbb{C}^{n+1}$, let $f : (\mathcal{U}, \mathbf{0}) \to (\mathbb{C}, 0)$ be an analytic function, and let $\Sigma f$ denote the critical locus of $f$. Let $L : \mathbb{C}^{n+1} \to \mathbb{C}$ be a generic linear form (we need for $L$ to be *prepolar* with respect to $f$; we shall define this in section 2).

The formula of Lê and Iomdine says that, if $\dim_{\mathbf{0}} \Sigma f = 1$, then, for all large $j$, $f + L^j$ has an isolated singularity at the origin and

$$b_n(f+L^j) \ = \ \mu(f+L^j) \ = \ b_n(f) - b_{n-1}(f) + j \sum_{\nu} m_\nu \delta_\nu(f),$$

where $b_i()$ denotes the $i$-th Betti number of the Milnor fibre of a function at the origin, $\mu$ denotes the Milnor number of the isolated singularity at the origin, the summation is over all components, $\nu$, of $\Sigma f$, $m_\nu$ is the local degree of $L$ restricted to $\nu$ at the origin, and $\delta_\nu(f)$ is the Milnor number of a generic hyperplane slice of $f$ at a point $\mathbf{p} \in \nu - \mathbf{0}$ sufficiently close to the origin.

This formula has, at least, two possible generalizations. One generalization is in terms of Lê numbers; this is the formula which appears in [**Mas1**], [**Mas3**], and [**M-S**]. But, while there are Morse inequalities between the Lê numbers and the Betti numbers of the Milnor fibre, the Lê numbers are not themselves (generally) Betti numbers of the Milnor fibre. So, one might ask