PACIFIC JOURNAL OF MATHEMATICS
Vol. 174, No. 2, 1996

EXPLICIT SOLUTIONS FOR THE CORONA PROBLEM WITH LIPSCHITZ DATA IN THE POLYDISC

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This paper contains considerations of various versions of the classical corona problem on domains in complex n-dimensional space. Although we do not solve the H^∞ corona problem, we do obtain positive results in other topologies. We also provide explicit constructions for solutions.

1. Introduction.

Let Δ be the unit disc in the complex plane, and let Δ^n be the unit polydisc in $\mathbb{C}^n$. We let $\mathcal{H}(\Delta^n)$ denote the space of all holomorphic functions on the polydisc, and $\mathcal{H}^p(\Delta^n)$ the holomorphic Hardy space on Δ^n (see [**Rud**]).For each $0 < \alpha < \infty$, we let $\Lambda_\alpha(\Delta^n)$ denote the holomorphic Zygmund spaces over Δ^n (see [**KR2**]). Suppose that $f_1, \ldots, f_m \in \mathcal{H}^\infty(\Delta^n)$ are such that

$$0 < \delta^2 \leq \sum_{j=1}^{m} |f_j(z)|^2 \leq 1, \qquad z \in \Delta^n. \tag{1.1}$$

In case $n = 1$, L. Carleson [**C**] solved the Corona problem and proved that there exist $g_j \in \mathcal{H}^\infty(\Delta)$ such that

$$\sum_{j=1}^{m} f_j(z) g_j(z) \equiv 1, \qquad \|g_j\|_{\mathcal{H}^\infty(\Delta)} \leq C(m, \delta).$$

The question of whether the Corona problem can be solved in several complex variables has attracted much attention (for example, see [**Am**], [**An**], [**AC**], [**Ch**], [**FS1, 2**], [**HS**], [**KL**], [**Li**], [**Lin**], [**S**], and [**V1, V2**], etc.). On a strongly pseudoconvex domain, there have been attempts to generalize the method of Hörmander [**H**] and of Wolff [**KO**] to higher dimensions. This entails solving a problem of the form $\bar\partial u = \mu$, with μ a Carleson measure. One seeks a bounded solution u. Such a bounded solution *does not always exist* when the dimension exceeds 1 (see [**V1**]). However it should be noted that the result of [**V1**] does *not* imply that the Corona problem fails in several variables—only that the $\bar\partial$ technique with *that particular definition of Carleson measure* fails.