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TWISTED ALEXANDER POLYNOMIAL AND REIDEMEISTER TORSION

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This paper will show that the twisted Alexander polynomial of a knot is the Reidemeister torsion of its knot exterior. As an application we obtain a proof that the twisted Alexander polynomial of a knot for an $SO(n)$-representation is symmetric.

Introduction.

In 1992, Wada [**4**] defined the twisted Alexander polynomial for finitely presentable groups. Let Γ be a finitely presentable group. We suppose that the abelianization $\Gamma/[\Gamma,\Gamma]$ is a free abelian group $T_r = \langle t_1, \dots, t_r | t_i t_j = t_j t_i \rangle$ of rank r. Then we will assign a Laurent polynomial $\Delta_{\Gamma,\rho}(t_1, \dots, t_r)$ with a unique factorization domain R-coefficients to each linear representation $\rho : \Gamma \to GL(n; R)$. We call it the twisted Alexander polynomial of Γ associated to ρ. For simplicity, we suppose that R is the real number field $\mathbf{R}$ and the image of ρ is included in $SL(n;\mathbf{R})$.

Because we are mainly interested in the case of the group of a knot, hereafter we suppose that Γ is a knot group. Let $K \subset S^3$ be a knot and E its exterior of K. We denote the canonical abelianization of Γ by

$$\alpha : \Gamma \to T = \langle t \rangle$$

and the twisted Alexander polynomial $\Delta_{\Gamma,\rho}(t)$ for $\Gamma = \pi_1 E$ by $\Delta_{K,\rho}(t)$. It is a generalization of the Alexander polynomial $\Delta_K(t)$ of K in the following sense. The Alexander polynomial $\Delta_K(t)$ of K is written as

$$\Delta_K(t) = (t-1)\Delta_{K,\mathbf{1}}(t)$$

where $\mathbf{1} : \Gamma \to \mathbf{R} - \{0\}$ is the 1-dimensional trivial representation of Γ.

On the other hand, Milnor [**2**] proved the following theorem about the connection between the Alexander polynomial and the Reidemeister torsion in 1962. We consider the abelianization

$$\alpha : \Gamma \to T$$