

## DERIVATIONS OF $C^*$ -ALGEBRAS AND ALMOST HERMITIAN REPRESENTATIONS ON $\Pi_k$ -SPACES

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**This paper studies almost Hermitian, J-symmetric representations of  $*$ -algebras on  $\Pi_k$ -spaces. It applies the results obtained to the theory of  $*$ -derivations  $\delta$  of  $C^*$ -algebras implemented by symmetric operators  $S$ .**

### 1. Introduction and preliminaries.

The work on representations of groups and algebras on spaces with invariant indefinite metric was strongly motivated by various applications to relativistic quantum mechanics and differential equations. Gelfand and Yaglom [9], Gelfand and Vilenkin [8], Naimark [27], Zhelobenko [44] and Ismagilov [12] investigated representations of the Lorentz group on  $\Pi_k$ -spaces. Representations of Lie groups were considered in a number of papers (see, for example, [4, 7, 10, 26]) in relation to the study of massless particles. The Gupta-Bleuer triplets for indecomposable representations of groups were introduced and studied by Araki [1]. Rawnslew, Schmid and Wolf [39] investigated the indefinite harmonic theory of groups. Ismagilov [13] considered representation theory on  $\Pi_k$ -spaces for central extension groups.

Phillips [34-36] initiated the work on operator algebras on indefinite metric spaces. He applied the obtained results to the problem of extension of dissipative operators commuting with an operator algebra and to hyperbolic systems of differential equations.

Simultaneously with the growth of the area of applications of the theory, the process of its internal development has been taking place. Much work has been done on the investigation of the structure of operator algebras and of representations of Lie groups on  $\Pi_k$ -spaces (for extensive bibliography on this subject see Naimark and Ismagilov [29], Naimark, Loginov and Shulman [31] and Loginov and Shulman [25]).

The interrelation between representations on indefinite metric spaces and unbounded  $*$ -derivations  $\delta$  of  $C^*$ -algebras  $\mathfrak{U}$  was initially observed by Ota [33] and by Jorgensen and Muhly [15]. Using Phillips' results, Jorgensen

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