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SOME PROPERTIES OF FANO MANIFOLDS THAT ARE ZEROS OF SECTIONS IN HOMOGENEOUS VECTOR BUNDLES OVER GRASSMANNIANS

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Let X be a Fano manifold which is the zero scheme of a general global section s in an irreducible homogeneous vector bundle over a Grassmannian. We prove that the restriction of the Plücker embedding embeds X projectively normal, and that every small deformation of X comes from a deformation of the section s. These results are strengthened in the case of Fano 4-folds.

Introduction.

Let $Gr(k,n) = \mathbf{SL}(n,\mathbb{C})/P_k$ be the Grassmannian of k-dimensional quotients of n-dimensional complex space $\mathbb{C}^n$ considered as quotient of $\mathbf{SL}(n,\mathbb{C})$ by a maximal parabolic subgroup P_k. Then (irreducible) representations of P_k give rise to (irreducible) homogeneous vector bundles over $Gr(k,n)$. The purpose of this note is to prove the following theorems:

Theorem 1. *Let X be a Fano manifold which is the zero scheme of a general global section in a globally generated irreducible homogeneous vector bundle $\mathcal{F}$ over $Gr(k,n)$. Then X is projectively normal.*

Here by a Fano manifold we mean a manifold X with ample anticanonical divisor $-K_X$, and $X \subset Gr(k,n)$ is considered to be embedded by the restriction of the Plücker embedding.

Theorem 2. *Let X be as above. Then every small deformation of X is again the zero scheme of a section in the same homogeneous bundle.*

Moreover it becomes obvious from the proof that the bundle $\mathcal{F}$ in Theorem 1 can be replaced by the sum of one irreducible vector bundle and line bundles.

Concerning Fano 4-folds we have a slightly more general result:

Theorem 3. *Suppose* $\dim(X) = 4$ *and that the Picard group* $\mathrm{Pic}(X)$ *of* X *is generated by* $\mathcal{O}_X(-K_X)$. *Then the statements of Theorems* 1 *and* 2 *remain*