PACIFIC JOURNAL OF MATHEMATICS
Vol. 175, No. 1, 1996

GLOBAL ANALYTIC HYPOELLIPTICITY OF $\Box_b$ ON CIRCULAR DOMAINS

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Let D be a smoothly bounded pseudoconvex domain in $\mathbb{C}^n$, $n \geq 2$, with real analytic boundary. In this paper we show that $\Box_b$ is globally analytic hypoelliptic if D is either circular satisfying $\sum_{j=1}^{n} z_j \frac{\partial r}{\partial z_j}(z) \neq 0$ near the boundary bD, where $r(z)$ is a defining function for D, or Reinhardt.

I. Introduction.

Let D be a smoothly bounded pseudoconvex domain in $\mathbb{C}^n$, $n \geq 2$, with real analytic boundary, and let $\mathbb{C}^n$ be equipped with the standard Euclidean metric. We consider the real analytic regularity problem of the $\Box_b$- equation on the boundary. Namely, given any $f \in C^{\omega}_{p,q}(bD)$, $0 \leq p \leq n-1$ and $1 \leq q \leq n-1$, let $u = N_b f \in L^2_{p,q}(bD)$ be the solution to the following equation,

$$\Box_b u = \left(\overline{\partial}_b \overline{\partial}_b^* + \overline{\partial}_b^* \overline{\partial}_b\right) N_b f = f. \tag{1.1}$$

Then we ask: is $u = N_b f \in C^{\omega}_{p,q}(bD)$? For the definitions of these notations the reader is referred to Section II.

The existence of the solution $u = N_b f$ is an immediate consequence of the closedness of the range of $\Box_b$ which was proved by M.C.Shaw [**17**] and Boas and M.C.Shaw [**1**], and independently by Kohn [**15**]. Since $u = N_b f$ is the canonical solution to the equation (1.1), it is unique. It also follows from Proposition 2.7. Next the real analyticity of the boundary bD implies that $u = N_b f$ is smooth, i.e., $u \in C^{\infty}_{p,q}(bD)$. For instance see Kohn [**14**][**16**]. Therefore, the main concern here is about the real analytic regularity of the solution u. The only result we know so far is that the answer is affirmative when D is of strict pseudoconvexity which is due to Tartakoff [**18**][**19**][**20**] and Treves [**21**] for $n \geq 3$ and to Geller [**13**] for $n = 2$.

The purpose of this article is to prove the following main results which presumably yield the first positive result to this problem on weakly pseudoconvex domains.