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THE CLOSED GEODESIC PROBLEM FOR COMPACT RIEMANNIAN 2–ORBIFOLDS

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In this paper it is shown that any compact Riemannian 2–orbifold whose underlying space is a (compact) manifold without boundary has at least one closed geodesic.

Introduction.

In this paper, we examine the question of the existence of a smooth closed geodesic on Riemannian 2–orbifolds. Roughly speaking a Riemannian orbifold is a metric space locally modelled on quotients of Riemannian manifolds by finite groups of isometries. It turns out that Riemannian orbifolds inherit a natural stratified length space structure and are sufficiently well–behaved locally so that one may apply both techniques of Alexandrov geometry and geometric analysis to extend standard results about Riemannian manifolds to Riemannian orbifolds. The 2–orbifolds we consider in this paper are orbifolds whose underlying space is a manifold *without* boundary. One can think of such Riemannian orbifolds as 2–manifolds with some distinguished singular cone points, whose neighborhoods are isometric to a quotient of the 2–disc with some metric by a cyclic group of finite order fixing the center of the disc. The 2–orbifolds we consider fall into two categories which we will handle with different techniques. The first case is when the underlying space of the orbifold is simply connected (in the usual topological sense), that is, the underlying space of the orbifold is the 2–sphere S^2. This class of orbifolds contains the set of all orientable *bad* 2–orbifolds, namely those that do not arise as a quotient of S^2 with some metric by a finite group of isometries acting properly discontinuously. These bad 2–orbifolds are examples of what are commonly referred to as teardrops and footballs. The second class of 2–orbifolds are those whose underlying space is not simply connected in the usual sense. The basic reference for orbifolds is [**T**], while a more Riemannian viewpoint is taken in [**B1**]. Many of the results on Riemannian orbifolds that we will use have appeared in published form in [**B2**].

Before we state and discuss our results for Riemannian orbifolds, we would like to recall the methods and ideas used to prove the classical theorem of Fet and Lyusternik [**FL**]: On any compact Riemannian manifold there