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ON THE STRUCTURE OF TENSOR PRODUCTS OF ℓ_p-SPACES

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We examine some structural properties of (injective and projective) tensor products of ℓ_p-spaces (projections, complemented subspaces, reflexivity, isomorphisms, etc.). We combine these results with combinatorial arguments to address the question of primarity for these spaces and their duals.

Introduction.

A Banach space X is *prime* if every infinite-dimensional complemented subspace contains a further subspace which is isomorphic to X. A Banach space X is said to be *primary* if whenever $X = Y \oplus Z$, X is isomorphic to either Y or Z. The classical examples of prime spaces are the spaces ℓ_p, $1 \leq p \leq \infty$. Many spaces derived from the ℓ_p-spaces in various ways are primary (see for example [**AEO**] and [**CL**]).

The primarity of $B(H)$ was shown by Blower [**B**] in 1990, and Arias [**A**] has recently developed further techniques which are used to prove the primarity of c_1, the space of trace class operators (this was first shown by Arazy [**Ar1**, **Ar2**]). It has become clear that these techniques are not naturally confined to a Hilbert space context; in the present paper we wish to extend the results to a variety of tensor products and operator spaces of ℓ_p-spaces (and in some cases $\mathcal{L}_p$-spaces). We also include some related results.

Some of the intermediate propositions (on factoring operators through the identity) may actually be true for a wider class of Banach spaces (those with unconditional bases which have nontrivial lower and upper estimates). In fact, the combinatorial aspects of the factorization can be applied quite generally, and may have other applications. The proofs of primarity, however, rely on Pełczyński's decomposition method which is not so readily extended. We have thus kept mainly to the case of injective and projective tensor products of ℓ_p spaces throughout. The results we obtain apply to the growing study of polynomials on Banach spaces since polynomials may be considered as symmetric multilinear operators with an equivalent norm (see [**FJ**], [**M**], or [**R**]).

Our main results are:

(1) If $1 < p < \infty$, then $B(\ell_p) \approx B(L_p)$.