PACIFIC JOURNAL OF MATHEMATICS
Vol. 175, No. 2, 1996

ON SPECTRA OF SIMPLE RANDOM WALKS ON ONE-RELATOR GROUPS

PIERRE-ALAIN CHERIX AND ALAIN VALETTE
WITH AN APPENDIX BY PAUL JOLISSAINT

For a one relator group $\Gamma = \langle X : r \rangle$, we study the spectra of the transition operators h_X and h_S associated with the simple random walks on the directed Cayley graph and ordinary Cayley graph of Γ respectively. We show that, generically (in the sense of Gromov), the spectral radius of h_X is $(\#X)^{-1/2}$ (which implies that the semi-group generated by X is free). We give upper bounds on the spectral radii of h_X and h_S. Finally, for Γ the fundamental group of a closed Riemann surface of genus $g \geq 2$ in its standard presentation, we show that the spectrum of h_S is an interval $[-r, r]$, with $r \leq g^{-1}(2g-1)^{1/2}$. Techniques are operator-theoretic.

1. Introduction.

Let Γ be a finitely generated group. Fix a finite, not necessarily symmetric generating subset X, and let $S = X \cup X^{-1}$ be the symmetrization of X. With X and S are classically associated the usual Cayley graph $G(\Gamma, S)$, but also the Cayley digraph (or directed graph) $G(\Gamma, X)$, where the set of vertices is Γ and, for any $\gamma \in \Gamma$ and $s \in X$, an oriented edge is drawn from γ to γs. We denote by $\#E$ the number of elements in the set E.

We consider the normalized adjacency operators, or transition operators, h_X and h_S; these are operators of norm at most 1 on $l^2(\Gamma)$, defined by:

$$(h_X\xi)(x) = \frac{1}{\#X} \sum_{s \in X} \xi(xs)$$

$$(h_S\xi)(x) = \frac{1}{\#S} \sum_{s \in S} \xi(xs) \qquad (\xi \in l^2(\Gamma), x \in \Gamma).$$

Consider the nearest neighbour simple random walk on $G(\Gamma, X)$ obtained by assigning probability $1/(\#X)$ to each neighbour of a given vertex $\gamma \in \Gamma$ (where a neighbour of γ is the extremity of an oriented edge with origin γ); then, for any $x, y \in \Gamma$, the probability $p^{(n)}(x, y)$ of a transition in n steps from x to y is given by $\langle h_X^n \delta_x | \delta_y \rangle$ (where $(\delta_x)_{x \in \Gamma}$ is the canonical basis of $l^2(\Gamma)$); the