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TENSOR PRODUCTS OF STRUCTURES WITH INTERPOLATION

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While it is known that the tensor product of two dimension groups is a dimension group, the corresponding problem for interpolation groups has been open for a while. We solve this problem here, by proving that the tensor product of two interpolation groups may not be an interpolation group, even for directed, torsion-free interpolation groups. We also solve the corresponding problems for refinement monoids (with tensor product of commutative monoids) and for lattice-ordered groups (with tensor product of partially ordered abelian groups).

0. Introduction.

Let A and B be two partially ordered abelian groups. Then the tensor product $A\otimes B$ (in the category of $\mathbb{Z}$-modules) can be given a structure of partially ordered abelian group, with positive cone the set of all sums $\sum_{i<n} a_i \otimes b_i$ where $n \in \mathbb{N}$ and for all $i < n$, $(a_i, b_i) \in A^+ \times B^+$ (this tensor product is related but *not* isomorphic to either kind of tensor product $A \otimes_o B$ or $A \otimes_\ell B$ considered in [**9**], where the result is always forced into being a ℓ-group even for arbitrary partially ordered abelian groups A and B). It is proven in [**5**] that the tensor product of two *dimension groups* (i.e. directed, unperforated partially ordered abelian groups with the interpolation property) is a dimension group. Then K.R. Goodearl asks in [**6**, Question 26] whether this holds for interpolation groups, i.e. whether the tensor product of two interpolation groups is an interpolation group.

We answer this question here, by giving several counterexamples where this does not hold (Examples 1.3 to 1.5), each of them with a specific feature. Our search for those counterexamples leads us first to study the connection between the positive cone of the tensor product of two partially ordered abelian groups and the tensor product of their positive cones as cancellative commutative monoids. Indeed, Example 1.3 shows that both are not necessarily isomorphic. Our constructions turn out in fact to be related to a counterexample of Manfred Dugas to [**6**, Question 2]. The common pattern between these counterexamples is that they show in particular that tensor