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HADAMARD-FRANKEL TYPE THEOREMS FOR MANIFOLDS WITH PARTIALLY POSITIVE CURVATURE

KATSUEI KENMOTSU AND CHANGYU XIA

In this paper we prove some theorems that two minimal submanifolds satisfying a condition for the dimensions of the submanifolds in a Riemannian manifolds with partially positive curvature or a Kaehler manifold with partially positive holomorphic sectional curvature must intersect. Our results show that the famous Frankel theorem about intersections of minimal submanifolds in a manifold with positive curvature is generalized to the very wide class of manifolds with partially positive curvature.

1. Introduction.

In 1966, Frankel [**F2**] showed that if N is an n-dimensional complete connected Riemannian manifold with strictly positive sectional curvature and if V is an r-dimensional compact totally geodesic immersed submanifold of N with $2r \geq n$, then the homomorphism of fundamental groups : $\pi_1(V) \to \pi_1(N)$ is surjective. This important theorem follows from an earlier result proved by himself in [**F1**] : Two compact totally geodesic submanifolds P and Q in a Riemannian manifold N of positive sectional curvature must necessarily intersect if their dimension sum is at least that of N. Unfortunately, the set of manifolds with positive sectional curvature is not so big. We don't even know whether the product of two 2-spheres $S^2 \times S^2$ admits a metric with positive sectional curvature or not.

Frankel also showed in [**F1**] that if N is a complete connected Kaehler manifold with positive sectional curvature, then any two compact analytic submanifolds must intersect if their dimension sum is at least that of N. Goldberg and Kobayashi proved in [**GK**] that the same conclusions also hold if N is only assumed to have positive bisectional curvature. We remark that since a complete connected Kaehler manifold of positive bisectional curvature which contains a compact Kaehler submanifold is compact (see Theorem 3.1 below), the above N is actually compact, thus it is biholomorphic to a complex projective space by the settlement of Frankel's conjecture (see [**M**] and [**SY**]). The topology of N is therefore very simple.