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SOLVABILITY OF DIRICHLET PROBLEMS FOR SEMILINEAR ELLIPTIC EQUATIONS ON CERTAIN DOMAINS

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We demonstrate a method to solve Dirichlet problems for semilinear elliptic equations on certain domains by a combination of change of variables, variational method and super-sub- solutions method. We show that Dirichlet problems for a semilinear elliptic equation have a least one solution as long as a relationship between the growth rate of the nonlinear term and the size of the domain is satisfied. The result can be applied to semilinear elliptic equations with super-critical growth.

1. Introduction and Results.

Let Ω be a bounded domain in R^n, $n > 2$. We consider the Dirichlet problem for a semilinear elliptic equation

$$(D_0) \qquad \begin{cases} -\Delta u = f(x,u) & \text{in} \quad \Omega; \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where Δ is the standard Laplace operator, $f(x,u)$ is a local Hölder continuous function defined on $\overline{\Omega} \times R$.

Throughout the paper, we assume that:

(†) There are positive constants M_1, M_2, $q \geq 1$, such that

$$|f(x,t)| \leq M_1 + M_2|t|^q \qquad \text{for all } \ x \in \overline{\Omega}, \ t \in R.$$

The main result of paper is

Theorem 1. *There is a constant $c(n,q)$ depending only on n and q, such that if we assume*

(1) (†);

(2) $|\Omega| \leq c(n,q)\left(M_2 M_1^{q-1}\right)^{-\frac{n}{2q}}$,