PACIFIC JOURNAL OF MATHEMATICS
Vol. 176, No. 1, 1996

UNITARY REPRESENTATION INDUCED FROM MAXIMAL PARABOLIC SUBGROUPS FOR SPLIT F_4

CHENG CHON HU (CHEN ZHONGHU)

For the linear connected simple Lie group split F_4, the author determines which Langlands quotients $J(MAN, \sigma, \nu)$ are infinitesimally unitary under the condition that $\dim A = 1$.

1. Introduction and Statement of Results.

It is known that the problem of classifying irreducible unitary representations of a linear connected semisimple Lie group G comes down to deciding which Langlands quotients $J(MAN, \sigma, \nu)$ are infinitesimally unitary. Here MAN is any cuspidal parabolic subgroup of G, σ is any discrete series or nondegenerate limit of discrete series representation of M, and ν is any complex valued functional on the Lie algebra of A satisfying $\operatorname{Re} \nu > 0$ and certain symmetry properties. Using Baldoni-Silva and Knapp [**BK3**], Baldoni-Silva and Knapp [**BK1**] determined which Langlands quotients are infinitesimally unitary under the conditions that G is simple, that $\dim A = 1$ and that G is neither split F_4 nor split G_2. Recently, the related problem was discussed by D.A. Vogan [**V3**] for the simply-connected split G_2. In this note, the author determines which Langlands quotients $J(MAN, \sigma, \nu)$ are infinitesimally unitary under the conditions that $\dim A = 1$ and that G is split F_4.

The author is deeply indebted to the referees and A.W. Knapp for their valuable opinions and their help. It is a great pleasure to acknowledege these.

Let G be the linear connected simple Lie group split F_4. Let θ be a Cartan involution, let K be the corresponding maximal compact subgroup, and let MAN be the corresponding Langlands decomposition of a parabolic subgroup. We shall assume that $\dim A = 1$. We denote corresponding Lie algebra by lower case Italy letters. Let σ be a discrete series representation of M or a nondegenerate limit of discrete series [**KZ2**], and let ν be a complex valued functional on the Lie algebra a of A. The standard induced representation $U(MAN, \sigma, \nu)$ is defined as in [**BK1**] (cf. p. 23 in [**BK1**]). If $\operatorname{Re} \nu \geq 0$ (with positive defined relative to N) and $\nu \neq 0$, then $U(MAN, \sigma, \nu)$ has a unique irreducible quotient $J(MAN, \sigma, \nu)$, the Langlands quotient. If ν is imaginary ,then $J(MAN, \sigma, \nu)$ is trivially unitary. If $\operatorname{Re} \nu > 0$, then