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MODULI SPACE OF ISOMETRIC PLURIHARMONIC IMMERSIONS OF KÄHLER MANIFOLDS INTO INDEFINITE EUCLIDEAN SPACES

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We classify isometric pluriharmonic immersions of a Kähler manifold into an indefinite Euclidean space. The moduli space of such immersions is explicitly constructed in terms of complex matrices. Some examples of these immersions are also given.

1. Introduction.

It has been a fundamental problem in the theory of minimal surfaces to determine the moduli spaces of isometric minimal immersions. An answer to this problem was given by Calabi [**3**], and recently it is generalized to higher dimensional cases by the present author [**8**]. In fact, we prove that the moduli space of isometric minimal immersions of a simply connected Kähler manifold into a *real* Euclidean space can be constructed in an explicit way as a set of certain complex matrices.

The purpose of this paper is to prove the counterpart of this construction in the case that the ambient space is an *indefinite* Euclidean space. Namely, we shall construct a parametrization of the moduli space of isometric pluriharmonic immersions of a simply connected Kähler manifold into an indefinite real Euclidean space in terms of certain complex matrices, which are in fact determined by a full isometric holomorphic immersion of the Kähler manifold into an indefinite complex Euclidean space. The key ingredient of our construction is the pluriharmonicity of these immersions, which eventually enables us to classify them in a similar fashion as in the case of minimal surfaces. However, it should be remarked that isometric maximal immersions of Kähler manifolds into indefinite Euclidean spaces are not pluriharmonic in general, which contrasts with the fact that isometric minimal immersions of Kähler manifolds into Euclidean spaces are always pluriharmonic.

In their paper [**1**, **2**], Abe-Magid proved a rigidity theorem of indefinite complex submanifolds and a representation formula for maximal surfaces in terms of holomorphic curves and complex matrices. Our result can be regarded as a sequel to their work.