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HEAT FLOW OF EQUIVARIANT HARMONIC MAPS FROM $\mathbb{B}^3$ INTO $\mathbb{CP}^2$

Y.L. XIN

We construct equivariant maps from $\mathbb{B}^3$ into $\mathbb{CP}^2$ and prove the global existence of heat flow of such equivariant harmonic maps for equivariant initial-boundary data which are not a priori required to have small range. We also show subconvergence of the solution. This supplies a regular harmonic extension of the given boundary condition.

1. Introduction.

The boundary value problem for harmonic maps has been studied by many mathematicians. For target manifolds with nonpositive sectional curvature, R. Hamilton [**H**] proved that such a boundary value problem is solvable by the heat flow method. In the case when the target manifolds have positive sectional curvature the situation becomes more complicated. If the boundary condition lies in a geodesic convex neighbourhood of the target manifold, S. Hildebrandt, H. Kaul and K.O. Widman [**H-K-W**] proved the existence of the boundary value problem by the direct method of the calculus of variations.

Although there exist examples to show the optimality of Hildebrandt-Kaul-Widman's theorem, one still expects the solvability for the boundary value problem with large image range when the boundary condition is "sufficiently nice". In [**J-K**] and [**E-L1**] the authors consider the rotationally symmetric harmonic maps from $\mathbb{B}^m$ into S^n whose boundary values lie just outside of a geodesic convex neighbourhood. Recently, many works have been written on maps from $\mathbb{B}^3$ into S^2 ([**Ha**], [**H-K-L1**], [**H-K-L2**], [**H-L-P**] and [**Z**]). Among them D. Zhang obtained a regular axially symmetric harmonic extension of $\mathbb{B}^3$ into S^2 for any regular axially symmetric boundary data which omit a neighbourhood of the south pole [**Z**]. As is well-known there are only two different kinds of isoparametric hypersurfaces in Euclidean space: umbilical ones and generalized cylinders. It is interesting to see that they correspond to the two kinds of reductions given by Jäger-Kaul and Zhang [**J-K**], [**Z**], respectively. By putting the problem in this framework with some essential technical improvement the result in [**Z**] has been improved in author's previous work [**X2**].