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IMBEDDING AND MULTIPLIER THEOREMS FOR DISCRETE LITTLEWOOD–PALEY SPACES

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We prove imbedding and multiplier theorems for discrete Littlewood–Paley spaces introduced by M. Frazier and B. Jawerth in their theory of wavelet-type decompositions of Triebel–Lizorkin spaces. The corresponding inequalities for discrete spaces defined in terms of characteristic functions of dyadic cubes, with respect to an arbitrary positive locally finite measure on the Euclidean space, are useful in the theory of tent spaces, weighted inequalities, duality theorems, interpolation by analytic and harmonic functions, etc. Our main tools are vector-valued maximal inequalities, a dyadic version of the Carleson measure theorem, and Pisier's factorization lemma. We also consider more general inequalities, with an arbitrary family of measurable functions in place of characteristic functions of dyadic cubes, which occur in the factorization theory of operators.

1. Introduction.

Let $\mathcal{Q} = \{Q\}$ be the family of all dyadic cubes in $\mathbf{R}^n$. Let ω be a nonnegative locally finite Borel measure on $\mathbf{R}^n$ such that $\int_{\partial Q} d\omega = 0$ for all $Q \in \mathcal{Q}$. We set $\mathcal{Q}_0 = \{Q \in \mathcal{Q} : |Q|_\omega \neq 0\}$, where $|Q|_\omega = \int_Q d\omega$; $|Q|$ will stand for the Lebesgue measure of Q. For any $Q \in \mathcal{Q}_0$, we denote by $\widetilde{\chi}_Q$ its normalized characteristic function $\widetilde{\chi}_Q = |Q|_\omega^{-1/2} \chi_Q$.

For $-\infty < \alpha < \infty$, $0 < p < \infty$, and $0 < q \le \infty$, we define the discrete Littlewood-Paley space $\mathbf{f} = \mathbf{f}_p^{\alpha q}(\omega)$ ([7], [8]) as the space of sequences of reals, $s = \{s_Q\}_{Q \in \mathcal{Q}_0}$, such that

$$(1.1) \qquad ||s||_{\mathbf{f}_p^{\alpha q}(\omega)} = \left\| \left(\sum_{Q \in \mathcal{Q}_0} \left(|Q|^{-\alpha/n} \, |s_Q| \, \widetilde{\chi}_Q \right)^q \right)^{1/q} \right\|_{L^p(d\omega)} < \infty.$$

Note that we use the normalized characteristic functions $\widetilde{\chi}_Q$ in the definition of $\mathbf{f}$ spaces in order to have the duality relation $[\mathbf{f}_p^{\alpha q}(\omega)]^* = \mathbf{f}_{p'}^{-\alpha q'}(\omega)$ with the usual pairing $\langle s, t \rangle = \sum s_Q \, t_Q$ $(s \in \mathbf{f}_p^{\alpha q}(\omega),\ t \in \mathbf{f}_{p'}^{-\alpha q'})$, at least for