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BRIDGED EXTREMAL DISTANCE AND MAXIMAL CAPACITY

ROBERT E. THURMAN[1]

We develop the concept of "bridged extremal distance" between disjoint sets X and Z on the boundary of a finitely connected domain G; that is, the extremal length of the family of curves connecting X and Z which are allowed to stop at a component of the "bridge" $Y = \partial G \setminus (X \cup Z)$ and re-emerge from any other point of that component. We connect bridged extremal distance with the extremal problem of "minimal extremal distance", and express it in terms of the period matrix associated with the harmonic measures of the boundary components of G. Then, in direct analogy to Ahlfors and Beurling's extremal length interpretation of logarithmic capacity, we use bridged extremal distance to give an extremal length interpretation of "maximal capacity".

1. Introduction.

Early in the development of extremal length, Ahlfors and Beurling related it to logarithmic capacity. Their survey article [**3**] introduced "reduced extremal distance" and connected it to capacity in the following way. Let Ω be a planar domain containing the point at infinity whose boundary Γ consists of a finite number of non-degenerate continua. Then Ω has a Green's function $g(z)$ with pole at infinity, and

$$\gamma = \lim_{z \to \infty} \{g(z) - \log |z|\}$$

is called *Robin's constant* for Ω. The *logarithmic capacity* of Γ is $d(\Gamma) = e^{-\gamma}$. To express γ in terms of extremal length, consider the domain $\Omega_R \subset \Omega$ bounded by Γ and the circle C_R centered at the origin of radius R, where R is large. Let λ_R denote the *extremal distance* between Γ and C_R; that is, the extremal length of the family of curves in Ω_R connecting Γ to C_R. Then λ_R increases to infinity with R, and Ahlfors and Beurling (see also [**2**],

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