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APPLICATIONS OF LOOP GROUPS AND STANDARD MODULES TO JACOBIANS AND THETA FUNCTIONS OF ISOSPECTRAL CURVES

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Let $L(z)$ be an element of $M_n(\mathbb{C}[z, z^{-1}])$. In this work we study the structure of isospectral curves given by $f(z,\lambda) = 0$, $f(z,\lambda) = \det(L(z) - \lambda)$, their Jacobians and the relationship between standard modules and the corresponding theta functions. We assume that $f(z,\lambda)$ is irreducible and nonsingular for $f(z,\lambda) = 0$ and $z \in \mathbb{C}^*$.

The element $L(z)$ will be called good, if the centralizers $\mathfrak{C}_\pm(L)$ of $L(z)$ in $M_n(\mathbb{C}[z])$ (resp. $M_n(\mathbb{C}[z^{-1}])$) are the integral closure of $\mathbb{C}[z, z^p L]$ (resp. $M_n(\mathbb{C}[z^{-1}, z^{-q}L])$) in $M_n(\mathbb{C}[z, z^{-1}])$. The class of curves we analyze include nonsingular curves and the isospectral curve of the periodic Toda lattice. The latter curve is represented by a "tridiagonal" matrix $L(z)$.

The Jacobian variety is expressed as a quotient of certain centralizers of $L(z)$ which are computed in a completion $M_n(A_w)$ of $M_n(\mathbb{C}[z, z^{-1}])$. If we assume further that $L(z)$ is an element of $\underline{SL}_n(\mathbb{C}[z, z^{-1}])$ then the basic module of the universal central extension $\widehat{SL}_n(A_w)$ of $SL_n(A_w)$ is employed to define a function Θ. This function Θ is defined in terms of representative functions on the "Lie theoretic" Jacobian and satisfies the functional equation of theta functions.

Introduction.

The relationship between completely integrable Hamiltonian systems, Kac-Moody Lie algebras and curve theory were studied systematically by M. Adler and P. van Moerbeke in [**1**], [**2**]. The main idea of their method is to associate to such a Hamiltonian system a Lax matrix differential equation of the form

$$\frac{dL}{dt} = [L, M(L)^+] = [L, M(L)^-]$$

where L is an element of a loop algebra $\tilde{\underline{g}} = \underline{g} \otimes \mathbb{C}[z, z^{-1}]$ and $M(L)$ is a function of L. The associated isospectral curve X_L is obtained as projective completion of the quasi-affine curve

$$X^a = \{(z,\lambda) \in \mathbb{C}^* \times \mathbb{C} \mid \det(L(z) - \lambda) = 0\}.$$