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On the fundamental groups of knotted 2-manifolds in the 4-space

By Takeshi YAJIMA

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1. Introduction

Let M be a 2-dimensional manifold imbedded in the 4-dimensional Euclidean space R^4. Let $\mathfrak{F}(M)$ be the fundamental group of R^4-M. In the case that M is a spinning sphere S, namely a sphere obtained by rotating an arc about a 2-dimensional plane, the group $\mathfrak{F}(S)$ was investigated by E. Artin [1], E. R. Van Kampen [2] and J. J. Andrews and M. L. Curtis [3].

The presentation of $\mathfrak{F}(S)$ was discussed by R.H. Fox [4] and S. Kinoshita [5], where S is a knotted 2-sphere in general. Their method, the so called moving picture method, concerned with the slice knots or the null-equivalent knots, which appear as an intersection of S and a 3-dimensional subspace of R^4.

This paper contains the method of the Wirtinger's presentation of $\mathfrak{F}(M)$ by the classical projection method as in the knot theory. In this direction the principle of the method has been given by S. Kinoshita [6].

As an application of this method, a parallelism between knots in R^3 and knotted 2-spheres in R^4 will be discussed.

2. Preliminaries

Let R^4 be the 4-dimensional Euclidean space with a coordinate system (x, y, z, u). Let R^3 be the 3-dimensional subspace of R^4 defined by $u=0$. With every point $P=(x, y, z, u)$ of a complex M in R^4, we associate the point $P^*=(x, y, z, 0)$ and $u=u(P)$. We call P^* the *trace* and u the *height* of a point P respectively and denote by $P=[P^*, u(P)]$. The set of traces of points of M will be denoted by M^*. The projection $\varphi: P\to P^*$ is defined as usual.

Throughout this paper terminologies are used in the semi-linear point of view. Hence complexes are polyhedral and mappings are simplicial.

Let M be a 2-dimensional closed orientable manifold. It is no loss of generality to assume the following condition:

(2.1) *If $P_1, \dots, P_m$ are vertices of M, then the system of points $(P_1^*, \dots, P_m^*)$ is in general position in R^3.*

Let $P^*\in M^*$. If there exist at least two points of M such that their traces