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Semi cubical theory on higher obstruction

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Let Y be a simply connected topological space which has vanishing homotopy groups $\pi_i(Y)$ for $0 \leqq i < n$, $n < i < q$, and $q < i < r < 2q-1$, and let K be a geometric complex with subcomplex L and $f: K^n \cup L \to Y$ be a mapping extensible to a map $K^{q+1} \cup L \to Y$. We discussed the third obstruction to the extension of f in [3].

It is the purpose of this paper to establish the higher obstruction theorems in the general cases by the aid of results of our preceding paper along the line of Eilenberg-MacLane [2]. This paper makes full use of the results and terminologies of the preceding paper of the author [4].

1. Preliminary

Let K and L are S.Q. complexes, we shall define the standard maps $f: K \times L \to K \otimes L$ and $g: K \otimes L \to K \times L$ between the cartesian and the tensor product. First map f is defined by

$$f(\sigma \times \tau) = \Sigma_\beta \beta_1^* \sigma \otimes \beta_2^* \tau \qquad \text{if } \dim \sigma = \dim \tau = r$$

where β is going round the family of pairs (β_1, β_2) such that

$$\beta_i : I^{m_i} \to I^r, \quad 0 \leqq m_i \leqq r, \; m_1 + m_2 = r,$$
$$\beta_1(t_1, \cdots, t_{m_1}) = (t_1, \cdots, t_{m_1}, 0, \cdots, 0),$$
$$\beta_2(t_1, \cdots, t_{m_2}) = (1, \cdots, 1, t_1, \cdots, t_{m_2}),$$

namely $\beta_1^* = F^{0 \cdot m_2} = F^0_{m_1+1} \cdots F^0_r$ and $\beta_2^* = {}^{m_1}F^1 = F^1_0 \cdots F^1_{m_1}$. Second map g is defined by

$$g(\sigma \otimes \tau) = \Sigma_\alpha \wp(\alpha) \alpha_1^* \sigma \times \alpha_2^* \tau \qquad \text{if } \dim \sigma = m_1, \; \dim \tau = m_2$$

where α is going round the family of pairs (α_1, α_2) such that

$$\alpha_i : I^r \to I^{m_i}, \quad r = m_1 + m_2,$$
$$\alpha_1(t_1, \cdots, t_r) = (t_{i_1}, \cdots, t_{i_{m_1}}) \qquad i_1 < \cdots < i_{m_1},$$
$$\alpha_2(t_1, \cdots, t_r) = (t_{j_1}, \cdots, t_{j_{m_2}}) \qquad j_1 < \cdots < j_{m_2},$$

and

$$\wp(\alpha) = \mathrm{Sgn.}\begin{pmatrix} 1, & \cdots\cdots\cdots\cdots\cdots\cdots, & r \\ i_1, \cdots, i_{m_1}, & j_1, \cdots, & j_{m_2} \end{pmatrix}.$$

LEMMA 1.1. *If K and L are S.Q. complexes, then each of the composites fg and gf is chain homotopic to the appropriate identity map.*

The proof of this lemma is similar to that of Eilenberg-Zilber theorem [1] in the S.S. complexes, and therefore we omit it.