Journal of the Institute of Polytechnics,
Osaka City University,
Vol. 11, No. 1, Series A

The structure of rings whose quotient rings are primitive rings with minimal one sided ideals

By Manabu HARADA

(Received October 22, 1959)

Recently A. W. Goldie [2] has proved that the quotient ring of a prime ring with some ascending chain condition is a simple ring with minimal condition. In this note we shall show that we can obtain the properties of a ring whose quotient ring is a primitive ring with minimal one sided ideals (P.M.I.), which are analogous to those of a prime ring in [2]. The following example shows that there exists such a ring.

Let I be the ring of rational integers. Let R_n be a sub-ring of matrix ring with infinite degree over the ring of rational numbers such that

$$\begin{pmatrix} (a_{ij}) & & & \\ & 2m_1 & & \\ & & 2m_2 & \\ & & & \ddots \end{pmatrix} \quad m_i \in I, \quad (a_{ij}) \in I_n .$$

Let $R=\bigcup_n R_n$, then if an element a of R is not zero divisor, a is the following form:

$$a=\begin{pmatrix} (a_{ij}) & & & \\ & 2m_1 & & \\ & & 2m_2 & \\ & & & \ddots \end{pmatrix} \quad |a_{ij}| \neq 0, \quad m_i \neq 0 .$$

Hence the right (and left) quotient ring of R is $Q=\bigcup Q_n$:

$$Q_n=\begin{pmatrix} (a_{ij}) & & & \\ & m_1 & & \\ & & m_2 & \\ & & & \ddots \end{pmatrix} \quad (a_{ij}) \in Q_n \quad \text{and} \quad m_i \in Q' ,$$

where Q' is the ring of rational numbers, and Q is P.M.I..

In this note there are many statements which overlap [2], but we shall repeat those for the sake of completeness.

1. Preliminaries.

Let R be a ring with the right and left quotient ring Q and we shall call non zero divisor elements regular elements. We shall denote one sided ideals of R by Roman and ones of Q by German.

We have the following statements.