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On the foundation of balayage theory

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Introduction

In this paper, we intend to show a new construction of the balayage (sweeping out) of measures which is one of the central themes in the potential theory. Our main tool used here is the noted theorem of Krein-Milman in the theory of general linear topological spaces (see [2], [7], etc). Thus, we begin with some detailed considerations about a linear normed space $H(D)$ and its dual $(H(D))^*$, especially some compact convex subset $\overline{M}_0^+(\overline{D})$ in $(H(D))^*$ generated by the collection of positive measures of norm 1 distributed in the closure of considered open set D (§**1**). The next paragraph (§**2**) is devoted to the general construction of balayage for open sets, but the same method is also well applicable to the case of closed sets, which is identical with the notion of so-called extremisation owing to M. Brelot (§**3**).

Now, from a historical point of view, the balayage theory founded by H. Poincaré has been recently reconstructed by means of projection method in the theory of Hilbert space; the most important work of such a kind is appeared in H. Cartan [4], and some interesting works of H. Cartan-J. Deny and of J. Deny follow it. However, in our present work, it seems very interesting that we can find some notable connection between the extreme points of $\overline{M}_0^+(\overline{D})$ (or of $\overline{M}_0^+(F)$) and regular (or resp. stable) boundary-points (§**4**), and as applications of this fact, we shall offer an elementary criterion in order that a boundary-point be regular or stable (Theorem 17).

§**5** is devoted to the representation theory and application to Dirichlet's problem in the ordinary or extended form. To obtain the solution, we employ the Banach space method here; thus, we are standing in some different position from the others.

The same theme of this paper has appeared incompletely in the last half of my previous note [10], and the present one is the precision and correction of that. However, we leave the general notion of superharmonicity to be defined as in the first half of [10].

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