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A note on Hattori's theorems

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Recently Hattori [3] has given a characterization of Prüfer rings. In this short note we shall show that this characterization is valid for a commatative ring with a slightly weaker property than that of an integral domain.

Let Λ be a commutative ring and S be a set of non zero divisors which possesses the following properties:

1) $S \ni 1$; 2) S is closed under multiplication.

We shall denote Λ_s the ring of quotients of Λ with respect to S. We shall call an element a of a Λ-module A a *torsion element* if $sa=0$ for some $s \in S$. The torsion elements form a submodule tA of A. We can define a *torsion-free module, divisible element*, etc. similarly to the case of an integral domain.

We can easily obtain the following results:

1) *we have an exact sequence*

$$0 \to tA \to A \to A_s = A \underset{\Lambda}{\otimes} \Lambda_s$$

for any Λ-module A,

2) *Λ_s is Λ-flat,*

3) *$w.\dim_\Lambda A_s = w.\dim_{\Lambda_s} A_s$ for any Λ-module A,*

4) *we have*

$$\{Tor_n^\Lambda(A, C)\}_s \approx Tor_n^{\Lambda_s}(A_s, C_s)$$

for any Λ-modules A and C. (*See Cartan and Eilenberg* [1], VII, *Exer's* 9 *and* 10).

From now on we shall always assume that Λ_s is *regular*.

For instance, an integral domain has this property. Let Λ be a complete direct sum of integral domains, then Λ_s is regular, where S is the set of non-zero divisors.

PROPOSITION 1. *Let A be a torsion-free, divisible Λ-module. Then $w.\dim_\Lambda A$ $=0$.*

Proof. If A is torsion-free and divisible, we can regard A as a Λ_s-module by the following definition.

For $\frac{\mu}{\lambda} \in \Lambda_s (\lambda \in s)$, and $a \in A$, there exists a unique element b in A such that $\lambda b = a$. We define $\frac{\mu}{\lambda} \cdot a = \mu b$. From equalities: $\frac{\mu}{\lambda} \cdot a = \mu\left(\frac{1}{\lambda} a\right) = \frac{1}{\lambda}(\mu a)$ we can prove that A is a Λ_s-module. By an inclusion mapping: $\Lambda \to \Lambda_s$ and 2) we have