Journal of the Institute of Polytechinics,
Osaka City University,
Vol. 6, No. 1, Series A

On the cell structure of the octanion projective plane Π

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(Received March 15, 1955)

1. Introduction.

As the projective plane, we have four main examples, namely, the real, complex, quaternion and octanion projective planes, which are denoted by P, M, Q and Π respectively. Since the real, complex, and quaternion number fields are associative, the projective planes over these fields can be treated by the quite similar methods and have the similar properties. However, since the octanion number field (*i. e.* Cayley number field) is non-associative, we can not sometimes treat it as well as the formers. H. Freudenthal [2], [3] and the others [1] firstly have constructed the octanion projective plane by the matrix method. This construction is also applicable in the associative cases. Our first purpose of this note is to give the connection between the usual construction and the above construction of P, M and Q (see §6).

The topological properties of the spaces P, M and Q are well known as the space with the simplest structure, for example, they are CW-complex[1)] in the sense of J. H. C. Whitehead [6], [7]. The second purpose of this note is to show that Π *is also a CW-complex in which the 16-dimensional cell* e^{16} *is attached to the 8-dimensional sphere* S^8 *by the Hopf map* $\eta: S^{15}\to S^8$ (see §4).

2. Definition of the octanion projective plane Π.

In §§2–5, the latin letters $x, y, z, s, t, \cdots\cdots$ will denote the octanion numbers and the Greek letters $\xi, \eta, \zeta, \sigma, \tau, \cdots\cdots$ the real numbers.

Let $\mathfrak{J}$ be the set of all hermitian matrices of three order

$$X=\begin{pmatrix}\xi_1 & x_3 & \bar{x}_2\\ \bar{x}_3 & \xi_2 & x_1\\ x_2 & \bar{x}_1 & \xi_3\end{pmatrix}^{2)}$$

with coefficients in the octanion number field. We define the multiplication in $\mathfrak{J}$ by

$$X\circ Y=\frac{1}{2}(XY+YX),$$

where XY is the usual matrix multiplication of X and Y. Then $\mathfrak{J}$ becomes a 27-dimensional distributive, commutative and non-associative algebra over the real

1) For the definition of CW-complex, see [8].

2) The symbols $\bar{x}$ and $|x|$ indicate the conjugate number and the absolute value of x respectively.