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EXTENSION OF MEASURES TO INFINITE DIMENSIONAL SPACES OVER *P*-ADIC FIELD

KUMI YASUDA

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1. Introduction

In carrying out analysis on infinite dimensional spaces over p-adics, it is useful to give integral representations of functions. Satoh considered a normed vector space H over a local field K with orthonormal Schauder basis ([14]). He showed that any admissible probability measure on K is extended to a measure on the completion of H with respect to a measurable norm, applying Prokhorov's measure extension theorem to the projective limit of the images of orthogonal projections on H. This can be applied to a space of polynomials with coefficients in p-adics. On the other hand the present paper aims at extending probability measures to spaces including extension fields over p-adics of infinite degree, in which there exist no orthonormal basis in the sense of [14], except the case of unramified extensions. The spaces to which we extend measures are completions of infinite extension fields over p-adics with respect to specific seminorms induced by projections naturally related with traces on subextensions. We notice that our projections are not necessarily orthogonal in the sense of [14]. The subjects of our theorem include for instance the algebraic closure and the maximal unramified extension of the p-adic field. Kochubei proved independently that Gaussian measures on a local field can be extended to completion of an infinite extension and constructed a fractional differentiation operator relative to the measure ([9]).

Let p be a fixed prime integer. The p-adic field $\mathbb{Q}_p$ consists of formal power series

$$\sum_{i=m}^{\infty} \alpha_i p^i, \quad m \in \mathbb{Z}, \ \alpha_i \in \{0, 1, \dots, p-1\}.$$

With ordinary addition and multiplication as power series, $\mathbb{Q}_p$ becomes a field. The p-adic norm $\|\cdot\|$ is defined by

$$\left\| \sum_{i=m}^{\infty} \alpha_i p^i \right\| = p^{-m} \quad \text{if } \alpha_m \neq 0, \quad \text{and} \quad \|0\| = 0.$$

We denote by $\mathbb{Z}_p$ the valuation ring $\{x \in \mathbb{Q}_p \mid \|x\| \leq 1\}$.