T'joën, L.
Osaka J. Math.
37 (2000), 925–951

UNIQUENESS IN THE CAUCHY PROBLEM FOR QUASI-HOMOGENEOUS OPERATORS WITH PARTIALLY HOLOMORPHIC COEFFICIENTS

LAURENT T'JOËN

(Received January 27, 1999)

1. Introduction and main reults

The purpose of this work is to extend to the case of quasi-homogeneous symbols the recent results of Tataru [10], Hörmander [3] and Robbiano-Zuily [7] concerning the uniqueness of the Cauchy problem for operators with partially holomorphic coefficients. Even in the merely C^∞ coefficients case our results will be more general that those given in Isakov [4], Dehman [1] and Lascar-Zuily [6]. The method used here will be basically the same as in the proof given by [7], that is the use of the Sjöstrand theory of FBI transform to microlocalize the symbols and then symbolic calculus for anisotropic pseudo-differential operators and the Fefferman-Phong inequality.

Let us be more precise. Let n, d be two non negative integers with $n+d \geq 1$. We shall set $\mathbb{R}^{d+n} = \mathbb{R}^d \times \mathbb{R}^n$ and, for X or ζ in $\mathbb{R}^{d+n}$, $X = (x, y)$, $\zeta = (\xi, \tau)$. Here y will be the "C^∞ variables" and x the "analytic ones".

Let $m = (m_1, \dots, m_n)$, $\tilde{m} = (\tilde{m}_1, \dots, \tilde{m}_d)$ be multi-indices, such that

$$(1.1) \qquad \begin{cases} 0 < m_1 \leq \cdots \leq m_{q-1} < m_q = \cdots = m_n = M, \\ 0 < \tilde{m}_1 \leq \cdots \leq \tilde{m}_{p-1} < \tilde{m}_p = \cdots = \tilde{m}_d = \tilde{M} = M. \end{cases}$$

We set $h_j = M/m_j$, $\tilde{h}_j = M/\tilde{m}_j$. $\{\cdot, \cdot\}_0$ will denote the quasi-homogeneous Poisson bracket that is

$$(1.2) \qquad \{f, g\}_0 = \sum_{j=q}^{n} \Big(\frac{\partial f}{\partial \tau_j} \frac{\partial g}{\partial y_j} - \frac{\partial f}{\partial y_j} \frac{\partial g}{\partial \tau_j} \Big) + \sum_{j=p}^{d} \Big(\frac{\partial f}{\partial \xi_j} \frac{\partial g}{\partial x_j} - \frac{\partial f}{\partial x_j} \frac{\partial g}{\partial \xi_j} \Big).$$

If $\alpha = (\alpha_1, \dots, \alpha_d) \in \mathbb{N}^d$, $\beta = (\beta_1, \dots, \beta_n) \in \mathbb{N}^n$, we set

$$(1.3) \qquad |\alpha : \tilde{m}| = \sum_{j=1}^{d} \frac{\alpha_j}{\tilde{m}_j}, \quad |\beta : m| = \sum_{j=1}^{n} \frac{\beta_j}{m_j}.$$