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LAWS OF THE ITERATED LOGARITHM FOR SOME SYMMETRIC DIFFUSION PROCESSES

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1. Introduction

In this paper we obtain three results concerning laws of the iterated logarithms (LILs) for certain functionals of some Markov processes.

The first is for symmetric diffusions whose transition densities satisfy upper and lower bounds similar to those of Aronson for uniformly elliptic divergence form operators in $\mathbf{R}^d$. We suppose the transition densities $p_t(x, y)$ are symmetric in x and y and satisfy an estimate of the form

$$\begin{aligned} &c_1 t^{-d_s/2} \exp\Big(-c_2\Big(\frac{d(x,y)^{d_w}}{t}\Big)^{1/(d_w-1)}\Big) \\ \le\ & p_t(x,y) \\ \le\ & c_3 t^{-d_s/2} \exp\Big(-c_4\Big(\frac{d(x,y)^{d_w}}{t}\Big)^{1/(d_w-1)}\Big), \end{aligned}$$

where $d(x, y)$ is the distance between x and y and c_1, c_2, c_3, c_4, d_s, and d_w are constants. Examples of such processes include ones associated to uniformly elliptic operators in divergence form in $\mathbf{R}^d$, of course, but also Brownian motions whose state space is an affine nested fractal, such as the Sierpinski gasket, and Brownian motions on Sierpinski carpets. (See the Appendix of this paper and also [14], [6] and [3].)

For such processes we first prove a large deviations principle similar to that of Schilder for Brownian motion (cf. [24]). When the state space is a fractal, one cannot prove as much as in the case of Brownian motion; in fact, it can be shown (see [10]) that the direct analog for Schilder's theorem is not true. Nevertheless, the large deviations principle that we do prove is sufficient to obtain a functional law of the iterated logarithm similar to that of Strassen; see Theorem 2.11. This is the content of Section 2.

Next in Section 3 we consider arbitrary Markov processes, not necessarily continuous nor symmetric, and look at functionals of the path that are nondecreasing, continuous, subadditive, and satisfy a uniform scaling property. For these functionals we

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