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ON THE GEOMETRIC INTERSECTION NUMBER OF AN IMMERSED MANIFOLD AND A PLANE

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1. Introduction and Main Theorem

Throughout this article, we work in the C^∞ category. Let $\mathbf{R}^N$ be the N-dimensional Euclidean space with the standard orientation and $f\colon M^{m_1} \to \mathbf{R}^N$ an immersion of an m_1-dimensional closed connected (possibly non-orientable) manifold, where $1 \leq m_1 < N$. We let m_2 denote the codimension of f ; $m_2 = N - m_1$. Let $P(N, m_2)$ be the set of oriented m_2-planes (lines if $m_2 = 1$) in $\mathbf{R}^N$. We will give a formula on the geometric intersection number $\#f^{-1}(f(M) \cap x)$ of the immersed manifold and an m_2-plane x in $\mathbf{R}^N$ in terms of algebraic intersection theory and "chamber-wall" structure on $P(N, m_2)$.

To the author's knowledge, the word "chamber" is used to indicate regions separated from a whole space by a codimension 1 subspace, for example, as "Weyl chamber" in Lie algebra theory (for more recent usage, see [2]). The word "wall" is used for the separating subspace. Following the history, in this paper, we use the two words in the same meanings as above.

The set $P(N, m_2)$ admits a structure of an $(m_2 + 1)m_1$-dimensional C^∞ oriented manifold. In fact, it is homeomorphic to the total space of the orthogonal complement vector bundle to the tautological bundle over the corresponding real oriented Grassmannian manifold $G(N, m_2)$.

In the space $P(N, m_2)$, a subset consisting of m_2-planes tangent to the immersed manifold forms "walls" which decompose the space into some "chambers". By giving orientation to the walls, one may associate an number to each chamber by algebraic intersection theory. We will introduce a canonical orientation to the walls and will show that the number associated to a chamber in this way is equal to the geometric intersection number $\#f^{-1}(f(M) \cap x)$ of the immersed manifold and an m_2-plane x belonging to the chamber.

In the next section, from the immersion f, we will construct an $((m_2+1)m_1-1)$-

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