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A THREE-MANIFOLD INVARIANT VIA THE KONTSEVICH INTEGRAL

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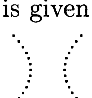
We construct an invariant for closed, oriented three-manifolds from the Kontsevich integral of framed links, and show that it includes Lescop's generalization of the Casson-Walker invariant. Combining this result and a formula for computing the Kontsevich integral in [17], we can compute the Casson-Walker invariant combinatorially in terms of q-tangles (non-associative tangles in [3]).

Our invariant is obtained from the Kontsevich integral by imposing the three-term (3T) relation, orientation independence (OI) relation, 0-vanishing relation and 1-vanishing relation to the space of chord diagrams subjected to the four-term relation. The *3T relation* is given by

(3T relation)  +  +  = 0.

Here, dotted lines present chords and the three chord diagrams are identical except within the region where they are as above. The *OI relation* is given as follows. Let D be a chord diagram and let D' be a chord diagram obtained by changing the orientation of a string s of D. Then

(OI relation) $$D' = (-1)^{e(s)}\, D.$$

Here $e(s)$ denotes the number of end points of chords on s. The *0-vanishing relation* means that a chord diagram having a string with no end points of chords is equal to 0, and the *1-vanishing relation* means that a chord diagram having a string with only one end point of chords is equal to 0.

The Kontsevich integral $\hat{Z}_f$ of a framed link has values in the space of chord diagrams subject to the four-term relation [13, 2, 17]. Let $\nu = \hat{Z}_f(\bigcirc)$ for the trivial knot $\bigcirc$, which is equal to the factor introduced in [2, 17] to normalize the effect of maximal and minimal points. For an ℓ-component oriented framed link L, let

$$\check{Z}_f(L) = \hat{Z}_f(L)\#(\nu, \nu, \cdots, \nu).$$

This means that we connect-sum ν to each string of $\hat{Z}_f(L)$. Let $\Lambda'(L)$ be the image of $\check{Z}_f(L)$ by the quotient of the space of chord diagrams by 3T, OI,