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A q-SERIES IDENTITY INVOLVING SCHUR FUNCTIONS AND RELATED TOPICS

Dedicated to Professor Takeshi Hirai on his sixtieth birthday

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1. Introduction

The main purpose of this paper is to prove:

Theorem 1.1. *For a Young diagram* $\lambda = (\lambda_1, \lambda_2, \lambda_3, \ldots)$, $s_\lambda(x) = s_\lambda(x_1, x_2, x_3, \ldots)$ *denotes the corresponding Schur function, and, for each node* v *in the diagram* λ, $h(v)$ *denotes the hook length of* λ *at* v. *Then we have the following identity with a parameter* q:

$$\sum_\lambda I_\lambda(q) s_\lambda(x) = \prod_i \prod_{r=0}^{\infty} \frac{1 + x_i q^{r+1}}{1 - x_i q^r} \prod_{i<j} \frac{1}{1 - x_i x_j}, \tag{1.1}$$

where

$$I_\lambda(q) = \prod_{v \in \lambda} \frac{1 + q^{h(v)}}{1 - q^{h(v)}}, \tag{1.2}$$

and the sum on the left of (1.1) *is taken over all Young diagrams* λ.

When $q = 0$, (1.1) reduces to the identity

$$\sum_\lambda s_\lambda(x) = \prod_i \frac{1}{1 - x_i} \prod_{i<j} \frac{1}{1 - x_i x_j} \tag{1.3}$$

due to Schur and Littlewood (see [12], I, 5, Ex. 4). On the other hand, when $x_1 = z$ and $x_2 = x_3 = \cdots = 0$, (1.1) reduces to the $t = q$ case of the q-binomial theorem

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