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35 (1998), 659–682

ON THE TRACE NORM ESTIMATE OF THE TROTTER PRODUCT FORMULA FOR SCHRÖDINGER OPERATORS

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(Received March 24, 1997)

0. Introduction

It is well-known that for the Schrödinger operator $(-1/2)\Delta + V$ with a nonnegative continuous potential V in the space $L_2(\mathbb{R}^d)$, the Trotter product formula

$$\lim_{n\to\infty}(e^{-tV/n}e^{-t(-(1/2)\Delta)/n})^n = e^{-t(-(1/2)\Delta+V)} \tag{0.1}$$

and its variant

$$\lim_{n\to\infty}(e^{-tV/2n}e^{-t(-(1/2)\Delta)/n}e^{-tV/2n})^n = e^{-t(-(1/2)\Delta+V)} \tag{0.2}$$

hold in the strong operator topology. It has recently been discussed that if V is e.g. in C^2 and satisfies

$$\begin{aligned} V(x) &\geq c(1+|x|^2)^{\rho/2} \\ |\nabla^m V(x)| &\leq c_m(1+|x|^2)^{(\rho-m)_+/2}, \quad m=1,2 \end{aligned} \tag{0.3}$$

for some $0 \leq \rho < \infty$, $0 < c < \infty$ and $0 \leq c_1,\ c_2 < \infty$ (which is the condition from [2]), (0.1) and (0.2) are convergent in the L_p-operator norm ($1 \leq p \leq \infty$). More precisely, as $t \downarrow 0$

$$\begin{aligned} &\|(e^{-tV/n}e^{-t(-(1/2)\Delta)/n})^n - e^{-t(-(1/2)\Delta+V)}\|_{p\to p} \\ &\qquad = \left(\frac{1}{n}\right)^{2/(2\vee\rho)} O(t^{1/2+1/(1\vee\rho)}) \end{aligned} \tag{0.4}$$

$$\begin{aligned} &\|(e^{-tV/2n}e^{-t(-(1/2)\Delta)/n}e^{-tV/2n})^n - e^{-t(-(1/2)\Delta+V)}\|_{p\to p} \\ &\qquad = \left(\frac{1}{n}\right)^{2/(2\vee\rho)} O(t^{1+2/(2\vee\rho)}), \end{aligned} \tag{0.5}$$

*Research partially supported by Grant-in-Aid for Scientific Research No. 09304022, Ministry of Education, Science and Culture, Japanese Government.