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FINITENESS THEOREMS FOR MEROMORPHIC MAPPINGS

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Introduction

The main purpose of this paper is to prove finiteness theorems for some families of meromorphic mappings that are transcendental in general. The finiteness problem for meromorphic mappings under the condition on the preimages of divisors was first studied by H. Cartan and R. Nevanlinna and they obtained a finiteness theorem for meromorphic functions on the complex plane $\mathbf{C}$ ([2] and [19]). The finiteness theorem of Cartan-Nevanlinna states that there exist at most two meromorphic functions on $\mathbf{C}$ that have the same inverse images with multi-plicities for distinct three values in $\mathbf{P}_1(\mathbf{C})$. In 1981, H. Fujimoto generalized the theorem of Cartan-Nevanlinna to the case of meromorphic mappings of $\mathbf{C}^m$ into complex projective spaces $\mathbf{P}_n(\mathbf{C})$ by making use of Borel's identity ([9], IV and [10]). He proved the finiteness of families of linearly nondegenerate meromorphic mappings of $\mathbf{C}^m$ into $\mathbf{P}_n(\mathbf{C})$ with the same inverse images for some hyperplanes. In his results, the number of hyperplanes in general position is essential and must be larger than a certain number depending on the dimension of the projective spaces. Furthermore, the finiteness theorem of Fujimoto has been extended to the case of meromorphic mappings into a projective algebraic manifold ([10] and [12]). In this paper, we mainly deal with the finiteness problem for meromorphic mappings f of $\mathbf{C}^m$ into a compact complex manifold M and for a divisor D on M.

Let $L \to M$ be a fixed line bundle over M, and let $\sigma_1, \cdots, \sigma_s$ be linearly independent holomorphic sections of $L \to M$ with $s \geq 2$. Throughout this paper, we assume that $(\sigma_j) = dD_j (1 \leq j \leq s)$ for some positive integer d, where D_j are effective divisors on M. Set

$$\varpi = c_1\sigma_1 + \cdots + c_s\sigma_s,$$

where $c_j \in \mathbf{C}^*$. Let D be a divisor defined by $\varpi = 0$. We define a meromorphic mapping $\Psi : M \to \mathbf{P}_{s-1}(\mathbf{C})$ by

$$\Psi = (\sigma_1, \cdots, \sigma_s).$$