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ON THE MODIFIED GOERITZ MATRICES OF 2-PERIODIC LINKS

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1. Introduction

An oriented link $\ell = k_1 \cup \cdots \cup k_\mu$ of μ components in S^3 is called a *2-periodic link* if there is a $\mathbb{Z}_2$-action on the pair (S^3, ℓ) such that the fixed point set f of the action is homeomorphic to a 1-sphere in S^3 disjoint from ℓ. It is known that f is unknotted. Hence the quotient map $p : S^3 \to S^3/\mathbb{Z}_2$ is an 2-fold cyclic branched covering branched over $p(f) = f_*$ and $p(\ell) = \ell_*$ is also an oriented link in the orbit space $S^3/\mathbb{Z}_2 \cong S^3$, which is called the *factor link* of ℓ.

In this paper, we express a relationship between the modified Goeritz matrices of a 2-periodic link ℓ and those of its factor link ℓ_* and the link $\ell_* \cup \bar{f}_*$. As an application, we give an alternative proof of the Gordon and Litherland's formular([3]): $\sigma(\ell) - Lk(\ell, \bar{f}) = \sigma(\ell_*) + \sigma(\ell_* \cup \bar{f}_*)$ for the signature $\sigma(\ell)$ of a 2-periodic null homologous oriented link ℓ in a closed 3-manifold M in the case of a 2-periodic oriented link in S^3. We also show that $n(\ell) = n(\ell_*) + n(\ell_* \cup \bar{f}_*) - 1$, where $n(\ell)$ denotes the nullity of an oriented link ℓ and $\bar{f}_*$ denotes the knot f_* with an arbitrary orientation.

2. Preliminaries

Let ℓ be an oriented link in S^3 and let L be its link diagram in the plane $\mathbb{R}^2 \subset \mathbb{R}^3 = S^3 - \{\infty\}$. Colour the regions of $\mathbb{R}^2 - L$ alternately black and white. Denote the white regions by $X_0, X_1, \cdots, X_w$ (We always take the unbounded region to be white and denote it by X_0). Let $C(L)$ be the set of all crossings of L. Assign an incidence number $\eta(c) = \pm 1$ to each crossing $c \in C(L)$ as in Fig. 2.1 and define a crossing $c \in C(L)$ to be of *type I* or *type II* as indicated in Fig. 2.1.

Let $S(L)$ denote the compact surface with boundary L, more precisely, $S(L)$ is built up out of discs and bands. Each disc lies in $S^2 = \mathbb{R}^2 \cup \{\infty\}$ and is a closed black region less a small neighbourhood of each crossing. Each crossing gives a small half-twisted band. Let $\beta_0(L)$ denote the number of the connected components of the surface $S(L)$.

Let $G'(L) = (g_{ij})_{0 \le i,j \le w}$, where $g_{ij} = -\sum_{c \in C_L(X_i, X_j)} \eta(c)$ for $i \neq j$ and $g_{ii} = \sum_{c \in C_L(X_i)} \eta(c)$, where $C_L(X_i) = \{c \in C(L) | c$ is incident to $X_i\}$ and $C_L(X_i, X_j) = \{c \in C(L) | c$ is incident to both X_i and $X_j\}$.