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ON A PROBLEM OF NAGATA RELATED TO ZARISKI'S PROBLEM*

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1. Introduction

Related to the problem proposed by Zariski[6] if the intersection $A \bigcap L$ of a normal affine ring A over a field k and a function field L over k is again an affine ring over k (we always understand that L is a subfield of a field containing A), Nagata obtained a characterization[3, Proposition 1], aiming at the affirmative answer, that the intersection $A \bigcap L$ of a normal affine ring A over a Dedekind domain k'(merely stated ground ring) and a function field L over k' is exactly an ideal transform of a normal affine ring over k'.

We recall that A is an affine ring over B if A is an integral domain containing B as a subring and is finitely generated over B and that L is a function field over B if L is the field of quotients of an affine ring over B.

Making use of this result, Rees constructed a counter example to Zariski's problem with an algebro-geometric consideration [5].

Recently, Nagata showed the following result[4, Theorem 2.1, 2.2], in view of the fact that the answer to Zariski's problem was negative and for generalizing the original results, where the derived normal ring of an integral domain A means the integral closure of A in its field of quotients.

Theorem 1.1 (Nagata). *Let B be a noetherian domain with the property $*$). Then the following on a ring R over B are equivalent.*
1) *The ring R has a form $\widetilde{A} \bigcap L$ with the derived normal ring $\widetilde{A}$ of an affine ring A over B and a function field L over B.*
2) *The ring R is the I-transform of the derived normal ring $\widetilde{C}$ of an affine ring C over B with an ideal I of $\widetilde{C}$.*

The property $*$) on B is the following,

$*$) *For every divisorial valuation ring D over B, the intersection $D \bigcap K$ of D*

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