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ASYMPTOTIC SOLUTIONS AND EXACT SOLUTIONS FOR EXCEPTIONAL CASES OF SOME CHARACTERISTIC CAUCHY PROBLEMS

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1. Introduction

M.S.Baouendi and C.Goulaouic([1]) considered a Fuchsian partial differential operator with weight $m-k$

$$P = t^k \partial_t^m + \sum_{l=1}^{k} b_{m-l}(x) t^{k-l} \partial_t^{m-l} + \sum_{j+|\alpha| \le m, j<m} t^{\max\{j-m+k+1,0\}} c_{j,\alpha}(t,x) \partial_t^j \partial_x^\alpha,$$

where m is a positive integer, k is a non-negative integer, $b_{m-l}(x)$ are holomorphic functions in a neighborhood of $x=0 \in \boldsymbol{C}^n$, and $c_{j,\alpha}(t,x)$ are holomorphic functions in a neighborhood of $(t,x)=(0,0) \in \boldsymbol{C} \times \boldsymbol{C}^n$. In the category of holomorphic functions, they showed the unique solvability of the characteristic Cauchy problem

$$\text{(CP)} \quad \begin{cases} Pu = f(x,t), \\ \partial_t^j u|_{t=0} = g_j(x) \quad (j=0,1,\ldots,\omega(P)-1) \quad (\omega(P) := m-k). \end{cases}$$

under the condition

$$\text{(A)} \qquad \mathcal{C}^{(P)}(0;\lambda) \neq 0 \quad \text{for} \quad \lambda \in \omega(P) + \boldsymbol{N} := \{\omega(P), \omega(P)+1, \ldots\},$$

where $\mathcal{C}^{(P)}(x;\lambda) := (\lambda)_m + \sum_{l=1}^{k} b_{m-l}(x)(\lambda)_{m-l}$ with $(\lambda)_j := \prod_{l=0}^{j-1}(\lambda - l)$. If the condition (A) is not satisfied, then the Cauchy problem does not necessarily have a holomorphic solution for every holomorphic Cauchy data. They also gave a similar result in the category of functions that are of C^∞ class in t and holomorphic in x.

The polynomial $\mathcal{C}^{(P)}(x;\lambda)$ of λ is called the *indicial polynomial* of P, and a root of $\mathcal{C}^{(P)}(x;\lambda)=0$ is called a *characteristic index* of P at x. A characteristic index λ is said to be *exceptional*, if $\lambda \in \omega(P) + \boldsymbol{N}$. The case when (A) is not satisfied, that is, when some characteristic indices at $x=0$ are exceptional, is called the *exceptional case.*