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SAMPLE LYAPUNOV EXPONENT FOR A CLASS OF LINEAR MARKOVIAN SYSTEMS OVER $\mathbb{Z}^d$

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1. Introduction

Let $\mathbb{Z}^d$ be the d-dimensional cubic lattice space, and let $\{Y_i(t)\}_{i\in\mathbb{Z}^d}$ be independent copies of a one-dimensional Lévy process $Y(t)$ defined on a probability space $(\Omega, \mathcal{F}, P^Y)$. Regarding $\{Y_i(t)\}_{i\in\mathbb{Z}^d}$ as random noises, we consider the following linear stochastic partial differential equation (SPDE) over $\mathbb{Z}^d$;

$$d\xi_i(t) = \kappa A\xi_i(t)dt + \xi_i(t-)dY_i(t) \quad (i \in \mathbb{Z}^d), \tag{1.1}$$

where $\kappa > 0$ is a constant and $A = \{a(i,j)\}_{i,j\in\mathbb{Z}^d}$ is an infinitesimal generator of a continuous time random walk on $\mathbb{Z}^d$, i.e.

$$a(0,i) \geq 0 \quad (i \neq 0), \quad \sum_{i\in\mathbb{Z}^d} a(0,i) = 0, \quad a(i,j) = a(0,j-i) \quad (i,j \in \mathbb{Z}^d) \tag{1.2}$$

and

$$A\xi_i = \sum_{j\in\mathbb{Z}^d} a(i,j)\xi_j.$$

Under a mild assumption on $Y(t)$ the equation (1.1) is well-posed and the solution defines a linear Markovian system in the sense of Liggett's book([7], Chap. IX).

When $\{Y_i(t)\}$ are independent copies of a standard Brownian motion, (1.1) is called *parabolic Anderson model* which has been extensively studied in [10], [8], [2], [3] from the view-point of intermittency. On the other hand when $Y_i(t) = -N_i(t) + t$ and $\{N_i(t)\}_{i\in\mathbb{Z}^d}$ are independent copies of a Poisson process with parameter one, (1.1) defines *a linear system with deterministic births and random deaths* introduced in [7], which was discussed from the view-point of ergodic problems. This process is a dual object of the survival probability of a random walker in a spatially and temporally fluctuating random environment, for which an asymptotic analysis was executed in [9]. The present form of the equation (1.1) was first treated in [1] where

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