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ON THE ABEL-JACOBI MAP FOR NON-COMPACT VARIETIES

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1. Introduction

Let X be a smooth projective variety over $\mathbb{C}$ of dimension n and S be a reduced normal crossing divisor on X. Then the generalized Jacobian $J(X-S)$ is a group $H^{n-1}(X,\omega_X(S))\check{}/H_{2n-1}(X-S,\mathbb{Z})$. When X is a curve, this fits into an exact sequence of algebraic groups:

$$1 \longrightarrow (\mathbb{C}^*)^{\sigma-1} \longrightarrow J(X-S) \longrightarrow J(X) \longrightarrow 0$$

where σ is the number of points in S and $J(X)$ is the usual Jacobian of X. Let $\mathrm{Div}^0(X-S)$ be the set of divisors of degree 0 on X which does not intersect with S. Then integration determines the Abel-Jacobi homomorphism $\alpha : \mathrm{Div}^0(X-S) \to J(X-S)$. We will prove an analogue of Abel's theorem (due to Rosenlicht [8] for curves) that the kernel of α is the following subgroup $\mathrm{Prin}_S(X)$ of S-principal divisors:

$$\mathrm{Prin}_S(X) = \{(f) \in \mathrm{Div}(X-S) | f \in K(X) \text{ and } f = 1 \text{ on } S\}.$$

A proof is a variation of our previous work [1], which involves reinterpretation of the Abel-Jacobi map in the language of mixed Hodge structures and their extensions. As a further application of this technique, we prove a Torelli theorem for a non-compact curve, which states that if X is the complement of at least 2 points in a nonhyperelliptic curve, then it is determined by the graded polarized mixed Hodge structure on $H^1(X,\mathbb{Z})$.

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2. Hodge Structures

DEFINITION 2.1. A (pure) Hodge structure H of weight m consists of a finitely generated abelian group $H_{\mathbb{Z}}$ and a decreasing filtration $F^\bullet$ of $H_{\mathbb{C}} := H_{\mathbb{Z}} \otimes \mathbb{C}$ such that $H_{\mathbb{C}} = F^p \oplus \overline{F^{m-p+1}}$.

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