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ON RIGIDITY OF HOLOMORPHIC MAPS OF RIEMANN SURFACES

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0. Introduciton

In this paper, we investigate holomorphic maps of Riemann surfaces using homology groups and free homotopty. There is a famous finiteness theorem concering holomorphic maps of compact Riemann surfaces.

Theorem 1 (de Franchis [3]). *Let $\tilde{X}$ be a compact Riemann surface of genus >1.*

(1) *For a fixed compact Riemann surface X of genus >1, the number of nonconstant holomorphic maps $\tilde{X}\rightarrow X$ is finite.*

(2) *There are only finitely many compact Riemann surfaces X_i of genus >1 such that there exists a nonconstant holomorphic map $\tilde{X}\rightarrow X_i$.*

The second assertion (2) is often attributed to Severi. For algebraic proofs of Theorem 1, see e.g. Kani [9], Martens [11][12], and Howard and Sommese [5]. Imayoshi [6][7] gave analytic proofs of these for Riemann surfaces of finite types.

Here we will study holomorphic maps of compact Riemann surfaces in terms of homology groups. We will show some rigidity theorems which guarantee Theorem 1. Let $\tilde{X}$, X be compact Riemann surfaces of genera $\tilde{g}$, $g(>1)$, and let $\{\tilde{\chi}_1,\cdots,\tilde{\chi}_{2\tilde{g}}\}$, $\{\chi_1,\cdots,\chi_{2g}\}$ be canonical homology bases on $\tilde{X}$, X, respectively. Let $h_i:\tilde{X}\rightarrow X$ be a nonconstant holomorphic map, and $M_i\in M\,(2g,2\tilde{g};\boldsymbol{Z})$ be a maxtrix representation of h_i $(i=1,2)$ with respect to $\{\tilde{\chi}_1,\cdots,\tilde{\chi}_{2\tilde{g}}\}$, $\{\chi_1,\cdots,\chi_{2g}\}$. Then, we will show

Theorem 2. *If there is an integer $l>\sqrt{8(\tilde{g}-1)}$ with $M_1\equiv M_2$ (mod. l), then $h_1=h_2$.*

In Theorem 2, the assumption concerns all of the entries of M_1, M_2. If we take l larger, we may assume conditions concerning merely a half number of entries of M_1, M_2 to get the same conclusion. For M_i, write

Holomorphic maps of Riemann surfaces