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Osaka J. Math.
32 (1995), 1087–1111

ISOTOPY CLASSES OF INCOMPRESSIBLE SURFACES IN IRREDUCIBLE 3-MANIFOLDS

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(Received April 25, 1994)

1. Introduction

Let M be a compact, orientable, irreducible, ∂-irreducible 3-manifold with a fixed triangulation $\mathscr{T}$. We are interested in the isotopy and projective isotopy classes of compact, orientable, incompressible, ∂-incompressible surfaces in M and how they are represented in the projective solution space $\mathscr{P}_{\mathscr{T}}$ from normal surface theory. Two surfaces belong to the same projective isotopy class if there exist multiples of each which are isotopic. We show that $\mathscr{P}_{\mathscr{T}}$ has maximal faces, called *complete lw-faces*, which have the following properties: (i) if a complete lw-face carries one surface in a projective isotopy class then it carries every least weight normal surface in that projective isotopy class and (ii) every surface carried by an lw-face is least weight in its isotopy class. Each complete lw-face is partitioned by compact linear cells in such a way that the set of surfaces carried by such a linear cell is precisely the set of all least weight surfaces in the corresponding projective isotopy class.

In normal surface theory there is associated to the triangulation $\mathscr{T}$ a system of matching equations whose admissible integral n-tuple solutions are in a one-to-one correspondence with the normal surfaces in M. The projections of such solutions to the unit sphere are called the *projective normal classes* of the corresponding normal surfaces and are contained in $\mathscr{P}_{\mathscr{T}}$, the compact, convex, linear cell of solutions to the normalized matching equations for $\mathscr{T}$. The *weight* of a normal surface F, denoted by wt(F), is the number of points in which F interesects the 1-skeleton of $\mathscr{T}$ and we say that F is a *least weight* surface if it is least wieght relative to its isotopy class. It has been shown in [2], [3], and [4] that least weight surfaces exhibit some strong and useful properties.

A face C of the compact, convex linear cell $\mathscr{P}_{\mathscr{T}}$ is said to be an *lw-face* if every normal surface carried by C is incompressible, ∂-incompressible and least weight. A *complete lw-face* C is an lw-face with the additional property that if a normal surface F is carried by C then every least weight normal surface isotopic to F is also carried by C. We show in Theorem 4.5 that there is a finite collection of complete lw-faces such that every compact, orientable, incompressible, ∂-