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SPEED LIMIT OPERATORS FOR OSCILLATING SPEED FUNCTIONS

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1. Introduction

As shown by Krengel and Lin [5], order preserving integral preserving operators in L_1^+ generalize the traditional model of Markov operators in such a way that interaction of the movement of mass particles is permitted. In the same paper, the study of the asymptotic properties of such operators was initiated, and speed limit operators for monotonely decreasing speed functions φ on the line were introduced, serving as examples and counterexamples. Roughly speaking, an integrable function $f \geq 0$ on the line describes a mass distribution. The mass particles move to the right. A given speed function $\varphi(x)$ assigns the highest permitted speed at the location x. However, particles can move more slowly if they are slowed down by slower particles in front of them.

In the present paper, we begin the study of speed limit operators for speed limits φ which need not be monotone. We assume that φ is piecewise constant and takes finitely many (at least two) different values. In a preliminary section we describe the evolution $T_t f$ of f when φ is monotonely increasing. In this case, there is no interaction, and T_t is linear. When φ oscillates, the mass is "spread out" in points of increase of φ, and points of decrease of φ cause interaction over possibly long distances. It is no longer possible to consider the movement of the various "levels" of the mass distribution separately.

The general non-monotonic case seems difficult. When φ is piecewise constant and the length of the intervals on which φ has a fixed value is assumed bounded below, an explicit recursive definition of $T_t f$ is possible for small $t > 0$ when f belongs to the class $\mathscr{F}$ of left-continuous piecewise constant nonnegative step functions. $T_t f$ again belongs to $\mathscr{F}$. The recursive definition works until there is a discontinuity in the "law of motion" of $T_t f$. A crucial step is to show that there are only finitely many

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