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LAGRANGEAN CONTACT STRUCTURES ON PROJECTIVE COTANGENT BUNDLES

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Introduction

Let (M,D) be a contact manifold of dimension $2n-1$, $n\geq 2$, and (E,E') a pair of subbundles of D. We say that $(D;\, E,E')$ is a Lagrangean contact structure on M if for each point $x\in M$ the fibres E_x and E'_x are transversal Lagrangean subspaces of D_x with respect to the natural conformal symplectic structure of D_x.

An example of Lagrangean contact structure is given on the projective cotangent bundle $P(T^*M)$ of a manifold M of dimension n in the following way. Let D be the canonical contact structure on $P(T^*M)$. Suppose that a projective structure Q on M is given. For $[\lambda]\in P(T^*M)$, we define $E'_{[\lambda]}$ to be the space of vertical vectors in $T_{[\lambda]}(P(T^*M))$ for the projection ϖ: $P(T^*M)\rightarrow M$. Furthermore, choosing a local torsionfree connection η belonging to Q defined over a neighbourhood of $x=\varpi([\lambda])\in M$, we define $E_{[\lambda]}$ to be the space of horizontal lifts to $[\lambda]$ of vectors $X\in T_xM$ with $\lambda(X)=0$. It is determined by Q independently on the choice of η. These subspaces $E_{[\lambda]}$, $E'_{[\lambda]}$ of $T_{[\lambda]}(P(T^*M))$, $[\lambda]\in P(T^*M)$, constitute subbundles E, E' of D such that $(D;\, E,E')$ becomes a Lagrangean contact structure on $P(T^*M)$ (Theorem 4.2).

A typical one is the Lagrangean contact structure $(D_0;\, E_0,E'_0)$ on the projective cotangent bundle of n-projective space P^n associated to the flat projective structure Q_0 on P^n. A Lagrangean contact structure is said to be flat if it is locally isomorphic to $(D_0;\, E_0,E'_0)$. The purpose of the present note is to prove:

The Lagrangean contact structure on $P(T^*M)$ associated to a projective structure Q on M is flat if and only if Q is projectively flat.

A conformal analogue to our theorem in the following form was proved by Miyaoka [2], Sato-Yamaguchi [3]: The Lie contact structure on the tangential sphere bundle $S(TM)$ associated to a conformal structure C on a manifold M is flat if and only if C is conformally flat, provided $\dim M\geq 3$.